Your name:

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Math 22 Fall 2016, Homework 9, due 8pm Tues, Nov 15

This one is slightly shorter since you have 1 day less.

(1) Let x_1 be the intercept and x_2 be the slope for a general linear function $y(t) = x_1 + x_2 t$. Find the least squares fit for this given the data $(0, 4)$, $(1, -2)$, and $(3, 0)$, which are three points (t, y) in the plane. Here's how to set up the linear system (you don't need to read Sec. 6.6 unless interested). The first point says $x_1 + x_2.0 = 4$, the next says $x_1 + x_2.1 = -2$, and the last says $x_1 + x_2.3 = 0$.

(a) Find the least squares solution vector(s) $\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$. Is the solution unique?

(b) What is the solution error? (Pythagorean sum of the y-errors between each data point and the line, ie your usual $\|\hat{\mathbf{b}} - \mathbf{b}\|$ error)

(c) Let A be any matrix, possibly rectangular. Prove that if $A^T A$ is invertible, then the columns of A are linearly independent.

(2) (a) Prove that, for any $m \times n$ matrix A, the set of nonzero eigenvalues of $A^T A$ is precisely the set of nonzero eigenvalues of AA^T. [Don't use the SVD. Hint: watch out for the case $A\mathbf{v} = 0$!]

(b) Consider the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Over all vectors $\mathbf{x}$ in $\mathbb{R}^3$ with $\|\mathbf{x}\| = 1$, what is the largest $\|A\mathbf{x}\|$ can be? [Hint: easier if exploit (a)]

(c) Compute by hand the full SVD of A, ie give U, Σ, and V. [Hints: you might find it easier to exploit that U is also the *right* singular vectors for A^T. Find the third column of V however you like.]