Your name:

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Math 22 Fall 2016, Homework 8, due Wed Nov 9

Please show your work. No credit is given for solutions without work or justification.

(1) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 5 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ and let $W = \text{Span}\,\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. In this question do not use row reduction.

(a) Use the fact that each vector in the set $\{\mathbf{v}_j\}$ is orthogonal to the other two to write $\mathbf{y} = \begin{bmatrix} 0 \\ 0 \\ -4 \\ 8 \end{bmatrix}$ as a linear combination of the three vectors.

(b) Write $W^{\perp}$ as the null space of a certain matrix.

(c) What is dim $W^{\perp}$? (Explain)

(2) (a) Compute the point in Span $\left\{ \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ which has the minimum distance from the point $\begin{bmatrix} 7 \\ 2 \\ 3 \end{bmatrix}$.

(b) What is the distance in part (a) ?

(c) Let A be any $n \times n$ matrix whose columns are an orthonormal basis for $\mathbb{R}^n$. Prove the amazing result that its rows must also be an orthonormal basis for $\mathbb{R}^n$. (As always with proofs, if you use previous theorems, explain how.)

(3) Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Use the Gram-Schmidt process to compute an orthogonal basis for Col A.

(b) Use the above to compute the factors Q and R in the QR factorization of A.