Your name:

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Math 22 Fall 2016, Homework 6, due Wed Oct 26

Please show your work. No credit is given for solutions without work or justification.

(1) (a) Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 0 & -2 & 4 \\ 1 & -1 & 6 \\ 1 & 0 & 4 \end{bmatrix}$. Find a basis for Row A:

(b) For the previous A, find a nontrivial vector that is guaranteed to have a zero dot product with every vector in Row A. In what fundamental subspace for the matrix A does such a vector live?

(c) What are the possible ranks of some 5×3 matrix B if there exists two nontrivial solutions to $B\mathbf{x} = \mathbf{0}$, neither of which is a multiple of the other?

(d) For a general matrix A, is the set Col A affected by row reductions?

BONUS. Is the dimension of Col A affected by row reductions?

(2) (a) For what h does the matrix $\begin{bmatrix} 11 & 1 \\ h & 3 \end{bmatrix}$ have an eigenvalue of (algebraic) multiplicity two? Show your work, as usual.

(b) For this h in the above matrix, what is the dimension of the eigenspace?

(c) For this h in the matrix from (a), explain why there can be no other eigenvalues.

(d) The vector $\mathbf{v} = (0, 0, 1)$ is an eigenvector of $\begin{bmatrix} -1 & -4 & 0 \\ 2 & 5 & 0 \\ 4 & 4 & 3 \end{bmatrix}$. Find another vector that together with $\mathbf{v}$ forms a basis for the eigenspace in which $\mathbf{v}$ lives:

(3) Find all eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ -5 & 3 & -3 \\ -4 & 0 & 0 \end{bmatrix}$. For each eigenvalue state its (algebraic) multiplicity, give a basis for its eigenspace and state the dimension of the eigenspace. [Hint: by noticing something you already know about the matrix, you can know one eigenvalue immediately. This helps you check your work.]