Your name:

Instructor (please circle): Alex Barnett Naomi Tanabe

Math 22 Fall 2016, Homework 5, due Wed Oct 19

Please show your work. No credit is given for solutions without work or justification.

(1) Consider the matrix $A = \begin{bmatrix} 1 & 3 & -9 & 3 \\ -1 & -1 & -3 & 1 \\ 2 & 3 & 0 & 0 \end{bmatrix}$

(a) Find a basis for Col A :

(b) Find a basis for Nul A :

(c) What is the dimension of the subspace spanned by the first three columns of A?

(d) For a *general* matrix A, do elementary row operations preserve Col A ? Nul A ?

(2) Let H be the subspace of vectors in $\mathbb{R}^3$ whose entries sum to zero.

(a) Each point in H is a unique combination of the linearly-independent vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$. Explain whether or not this set of three vectors is a basis for H.

(b) What is dim H ? Prove your answer.

(c) For a general vector space V with basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$, prove that the coordinate mapping $\mathbf{x} \mapsto [\mathbf{x}]_{\mathcal{B}}$ from V to $\mathbb{R}^n$ is one-to-one.

(3) Recall that $\{1, t, t^2\}$ form the standard basis for $\mathbb{P}_2$, the vector space of polynomials of degree at most two. In applications it is crucial to be able to handle polynomials expressed about a new origin (eg, for Taylor series). Let's shift the origin to the value $t = 3$.

(a) Prove that the set $\{1,\ t-3,\ (t-3)^2\}$ form a basis for $\mathbb{P}_2$. [Hint: you may use that the coordinate mapping to $\mathbb{R}^3$ is an isomorphism.]

(b) Say you are given the polynomial $a_0 + a_1 t + a_2 t^2$ but want to write it about the new origin, $b_0 + b_1(t-3) + b_2(t-3)^2$. Express each of the coefficients b_0, b_1, and b_2 in terms of a_0, a_1, and a_2.