Your name:

Instructor (please circle): Alex Barnett Naomi Tanabe

Math 22 Fall 2016, Homework 4, due Wed Oct 12

Please show your work. No credit is given for solutions without work or justification.

(1) (a) Compute the determinant of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 1 \\ 6 & 6 & 3 & 1 \\ 10 & 6 & 3 & 1 \end{bmatrix}$

(b) Is the matrix from (a) invertible, and why?

(c) Let A and B be arbitrary $n \times n$ matrices. Prove that $\det(BA) = \det(AB)$. (You may use results we have done, but state which.)

(2) Is each of the following sets a vector space? Explain what result(s) you used to prove your claim. You may assume that $\mathbb{R}^n$ is a vector space.

(a) $\left\{ \begin{bmatrix} t \\ 3 \end{bmatrix}, \quad t \text{ is real} \right\}$

(b) $\left\{ \begin{bmatrix} 0 \\ 3+t \end{bmatrix}, \quad t \text{ is real} \right\}$

(c) $\left\{ \begin{bmatrix} a+b \\ -a \\ 2a-b \end{bmatrix}, \quad a \text{ and } b \text{ are real} \right\}$

(d) All the vectors in $\mathbb{R}^4$ for which the first two entries are equal and the sum of all four entries is zero.

(3) True or false (no working needed, just circle the answer):

(a) T / F: $\det(A+B) = \det A + \det B$

(b) T / F: Given a square matrix, if you add three times row 2 to row 1, the determinant is multiplied by three.

(c) T / F: If you double every entry of a 3×3 matrix, its determinant is multiplied by 8.

(d) T / F: For any $m \times n$ matrix A, if $\text{Col } A = \mathbb{R}^m$ this implies $m \geq n$.

(e) T / F: For any $m \times n$ matrix A, and right hand side $\mathbf{b}$ in $\mathbb{R}^m$, the solution set of the linear system $A\mathbf{x} = \mathbf{b}$ is a subspace of $\mathbb{R}^n$.

(f) T / F: The set of all functions on $[0,1]$ with $f(\frac{1}{2}) = 0$ is a subspace of the vector space of all functions on $[0,1]$.