Your name:

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Math 22 Fall 2016, Homework 2, due Wed Sep 28

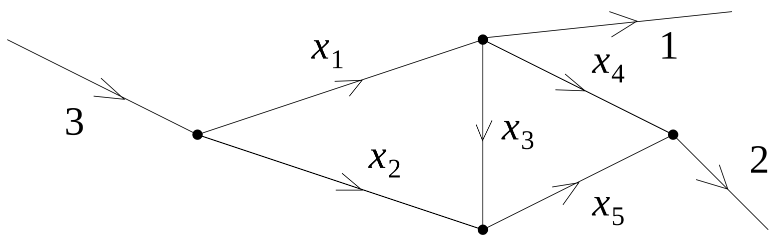
Please show your work, and check your answers. No credit is given for solutions without work or justification.

(1) Consider the set of three vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 6 \\ h \end{bmatrix}$.

(a) With $h = 0$, is the set linearly independent, and why? If not, give a dependence relation between $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

(b) With $h = 11$, is the set linearly independent, and why? If not, give a dependence relation between $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

(2) Consider the following network flow problem, where the numbers indicate known flows, and $x_1, \ldots, x_5$ unknown flows, on their respective edges:



(a) Find the *reduced* echelon form for the linear system corresponding to the flow problem:

(b) If a solution exists, write the *parametric vector form* of the solution set (include the numerical values of any vectors you use):

BONUS: In a general such network flow problem (where all flows to the outside world are known, and all interior flows unknown), what *geometric property of the network* does the number of free variables tell you? Why?

(3) Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be a set of n vectors in $\mathbb{R}^m$.

(a) Define $\text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, using clear verbal or mathematical language. (Careful: do not assume later theorems or properties of span!)

(b) If $n < m$, can it be that $\text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\} = \mathbb{R}^m$? Prove your answer (ie explain as clearly as you can).

(c) If $n = m$ and $\text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\} = \mathbb{R}^m$, what can you say about linear (in)dependence of the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$? Prove your answer.