Math 22 Fall 2013

Problem set 6: Due on Wed Nov 6

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. Show that if A is diagonalizable, and B is similar to A, then B is also diagonalizable.

2. Find the $\mathcal{B}$-matrix $[T]_{\mathcal{B}}$ for the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ whose standard matrix is the matrix A below, where $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$.

$$A = \begin{pmatrix} -6 & -2 \\ 4 & 0 \end{pmatrix},\ \mathbf{v}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

3. Let $L = \text{Span}\{\mathbf{b}_1\}$ be a line in $\mathbb{R}^3$, and $W = \text{Span}\{\mathbf{b}_2, \mathbf{b}_3\}$ a plane.

$$\mathbf{b}_1 = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix},\ \mathbf{b}_2 = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix},\ \mathbf{b}_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

   (a) Show that the line L is orthogonal to the plane W.

   (b) What is the closest point in the plane W to the vector $\mathbf{v} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$?

   (c) What is the closest point on the line L to the same vector $\mathbf{v}$?

   (d) Find the orthogonal decomposition $\mathbf{v} = \hat{\mathbf{v}} + \mathbf{z}$, where $\hat{\mathbf{v}}$ is a vector in W and $\mathbf{z}$ is a vector that is perpendicular to W.