

Math 22 Fall 2013

Homework 3: Due on Wed Oct 9

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the multivariable function given by the formula

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 - x_3, 3x_1 + 6x_2, x_1 + 2x_2, x_2 + 4x_3)$$

- (a) T is a linear transformation. What is the standard matrix A of T ?
 - (b) Is the transformation T one-to-one? Justify your answer.
 - (c) Is the transformation T onto? Justify your answer.
2. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation that corresponds to a rotation in space around the x_3 -axis by the angle $\pi/3$ (or 60°). We use the right-hand rule to determine the direction of the rotation.
 - (a) What is the standard matrix A of the transformation T ?
 - (b) Use the formula $T(\mathbf{x}) = A\mathbf{x}$ to calculate the image of the point $\mathbf{x} = (2, -3, 7)$ under the transformation T .
 3. Let A, B, C be the following matrices

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 1 \\ 0 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 2 & -1 \\ -2 & 0 \end{pmatrix}$$

- (a) Calculate AB, AC , and then add AB and AC to get $AB + AC$.
- (b) Calculate $B + C$ and then multiply A and $B + C$ to get $A(B + C)$.

Note. Since always $A(B + C) = AB + AC$, your final answer in item (b) should be the same as that in (a), but the calculations are different.
- (c) Is $CB = BC$ in this case?