Math 22 Fall 2013

Problem set 2: Due on Wed Oct 2

Show all your calculations. You can receive partial credit for partially correct work, even if the final solution is incorrect. Therefore, spell out step-by-step calculations, and explain your answers to open questions.

1. Let A be the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & 9 \\ 2 & 1 & 1 & 21 \\ -3 & 0 & -2 & -27 \end{pmatrix}$$

   Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution $\mathbf{x}$ in $\mathbb{R}^4$ for *every* vector $\mathbf{b}$ in $\mathbb{R}^3$?

   As always, show your numerical work, and justify your answer in words.

2. (a) Let A be the matrix,

$$A = \begin{pmatrix} 1 & 4 & -2 & 3 \\ 1 & 7 & 0 & 5 \\ 2 & 14 & 0 & 10 \\ -1 & -7 & 0 & -5 \end{pmatrix}$$

   Write the solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$ as a span of vectors.

   (b) Is the equation $A\mathbf{x} = \mathbf{b}$ consistent for *all* possible vectors $\mathbf{b}$ in $\mathbb{R}^4$?

   (c) Assuming that $\mathbf{b}$ is a vector in $\mathbb{R}^4$ for which $A\mathbf{x} = \mathbf{b}$ is consistent, how many free variables (or how many parameters) will there be in the description of the solution set?

3. Determine if the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ below are linearly independent.

$$\mathbf{u} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \ \mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \ \mathbf{w} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

4. True or False?

   For True/False questions you do not *have to justify your answer!*

   (a) If A is a 7×10 matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has a solution for all possible vectors $\mathbf{b}$ in $\mathbb{R}^7$.

   (b) If A is a 7×10 matrix, then the equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions.

   (c) If $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions then $A\mathbf{x} = \mathbf{b}$ cannot have a *unique* solution, no matter what choice you make for $\mathbf{b}$.

   (d) If 3 vectors in $\mathbb{R}^3$ lie in the same plane then they are linearly dependent.

   (e) If a set of vectors in $\mathbb{R}^4$ is linearly independent, then there are at least 4 vectors in the set.