Workshop IV

2006-10-26

1. Let A be an $m \times n$ matrix and B an $n \times p$ matrix.

   (a) Show that $\text{Nul } B \subseteq \text{Nul } AB$. What does this tell you about $\dim \text{Nul } B$ and $\dim \text{Nul } AB$?

   (b) Show that $\text{Col } AB \subseteq \text{Col } A$. What does this tell you about rank AB and rank A?

   (c) Show that $\text{rank } AB \leq \min\{\text{rank } A, \text{rank } B\}$; i.e., show that $\text{rank } AB \leq \text{rank } A$ and $\text{rank } AB \leq \text{rank } B$.

2. Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be a set of vectors in a vector space V. Prove that if for all $\mathbf{v} \in V$, the vector equation $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{v}$ has *at most* one solution, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a linearly independent set.