Workshop III

2006-10-19

1. Let V be a vector space and $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ a basis for V. Show that a subset $\{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ in V is linearly independent if and only if the set of coordinate vectors $\{[\mathbf{u}_1]_{\mathcal{B}}, \ldots, [\mathbf{u}_p]_{\mathcal{B}}\}$ is linearly independent in $\mathbb{R}^n$.

2. Let V and W be vector spaces, $T : V \to W$ a linear transformation, and $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ a subset of V.

   (a) Show that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent in V, then the set of images $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is linearly dependent in W.

   (b) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly independent, must $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ be linearly independent?