Products Involving Vectors

Scalar Multiplication: $k\mathbf{a}$ (valid in all dimensions)

Result	vector in the direction of $\mathbf{a}$
How to compute	$k(a_1, a_2, \ldots, a_n) = (ka_1, ka_2, \ldots, ka_n)$
	$k(a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \cdots + a_n\mathbf{e}_n) = (ka_1)\mathbf{e}_1 + (ka_2)\mathbf{e}_2 + \cdots + (ka_n)\mathbf{e}_n$
Magnitude	$\|\|k\mathbf{a}\|\| = \|k\|\ \|\|\mathbf{a}\|\|$
Zero if	$k = 0$ or $\mathbf{a} = \mathbf{0}$
Commutativity	Yes: $k\mathbf{a} = \mathbf{a}k$
Associativity	Yes: $k(l\mathbf{a}) = (kl)\mathbf{a}$
Distributivity	Yes: $k(\mathbf{a} + \mathbf{b}) = k\mathbf{a} + k\mathbf{b}$

Dot (Scalar) Product: $\mathbf{a} \cdot \mathbf{b}$ (valid in all dimensions)

Result	scalar
How to compute	$(a_1, a_2, \ldots, a_n) \cdot (b_1, b_2, \ldots, b_n) = a_1b_1 + a_2b_2 + \cdots + a_nb_n = \sum_{i=1}^{n} a_ib_i$
Magnitude	$\mathbf{a} \cdot \mathbf{b} = \|\|\mathbf{a}\|\|\ \|\|\mathbf{b}\|\| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$
maximized if	$\mathbf{a}$ is parallel to $\mathbf{b}$
Zero if	$\mathbf{a}$ is orthogonal to $\mathbf{b}$, or $\mathbf{a} = \mathbf{0}$, or $\mathbf{b} = \mathbf{0}$
Commutativity	Yes: $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
Associativity	Irrelevant: $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ makes no sense
Distributivity	Yes: $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
Properties	$\mathbf{a} \cdot \mathbf{a} = \|\|\mathbf{a}\|\|^2$

Cross (Vector) Product: $\mathbf{a} \times \mathbf{b}$ (valid in dimension $n = 3$ only)

Result	vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$
How to compute	$(a_1, a_2, a_3) \times (b_1, b_2, b_3) = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$
	$(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \times (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
Magnitude	$\|\|\mathbf{a} \times \mathbf{b}\|\| = \|\|\mathbf{a}\|\|\ \|\|\mathbf{b}\|\| \sin\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$
maximized if	$\mathbf{a}$ is orthogonal to $\mathbf{b}$
Zero if	$\mathbf{a}$ is parallel to $\mathbf{b}$, or $\mathbf{a} = \mathbf{0}$, or $\mathbf{b} = \mathbf{0}$
Commutativity	Anticommutative: $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
Associativity	No: In general $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) \neq (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$
Distributivity	Yes: $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$ and $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
Properties	If $\|\|\mathbf{a} \times \mathbf{b}\|\|$ is area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$
	If $\mathbf{a}$ is orthogonal to $\mathbf{b}$, then $\|\|\mathbf{a} \times \mathbf{b}\|\| = \|\|\mathbf{a}\|\|\ \|\|\mathbf{b}\|\|$

Mixed Products:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\left|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})\right|$ is volume of the parallelepiped spanned by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$