Math 22 Fall 2004

Linear Algebra with Applications

Vector and Matrix Equations

September 29, 2004

Load the package for doing Linear Algebra

```
> with(Student[LinearAlgebra]):
```

```
Warning, the protected name . has been redefined and unprotected
```

Example 1: Solve a vector equation

$$x1*\mathbf{a1} + x2*\mathbf{a2} = \mathbf{b}$$

```
> a1 := <-1, 3, -2>: a2 := <3, 2, -3>: b := <5, -4, 1>:
  a1, a2, b;
```

$$\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$$

Define an (augmented) matrix of the equation

```
> A := <a1 | a2 | b>;
```

$$A := \begin{bmatrix} -1 & 3 & 5 \\ 3 & 2 & -4 \\ -2 & -3 & 1 \end{bmatrix}$$

Reduce it to an echelon form

```
> A := AddRow(A, 2, 1, 3):
```

```
A := AddRow(A, 3, 1, -2);
```

$$A := \begin{bmatrix} -1 & 3 & 5 \\ 0 & 11 & 11 \\ 0 & -9 & -9 \end{bmatrix}$$

```
> A := MultiplyRow(A, 2, 1/11):
  A := AddRow(A, 3, 2, 9);
```

$$A := \begin{bmatrix} -1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

... and to the reduced echelon form

```
> A := AddRow(A, 1, 2, -3);
```

$$A := \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Solve the system now

```
> x := LinearSolve(A);
```

$$x := \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Test our solution

```
> x[1] * a1 + x[2] * a2 = b;
```

$$\begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$$

Example 2: Solve a matrix equation $A\mathbf{x}=\mathbf{b}$

```
> A := <<1, 3, -2> | <2, -1, 3> | <-2, -3, 1>>:
  b := Vector(3, symbol = v):
  M := <A | b>:
  A, b, M;
```

$$\begin{bmatrix} 1 & 2 & -2 \\ 3 & -1 & -3 \\ -2 & 3 & 1 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \begin{bmatrix} 1 & 2 & -2 & v_1 \\ 3 & -1 & -3 & v_2 \\ -2 & 3 & 1 & v_3 \end{bmatrix}$$

Solve this system

```
> M := AddRow(M, 2, 1, -3):
  M := AddRow(M, 3, 1, 2);
```

$$M := \begin{bmatrix} 1 & 2 & -2 & v_1 \\ 0 & -7 & 3 & v_2 - 3\,v_1 \\ 0 & 7 & -3 & v_3 + 2\,v_1 \end{bmatrix}$$

```
> M := AddRow(M, 3, 2, 1);
```

$$M := \begin{bmatrix} 1 & 2 & -2 & v_1 \\ 0 & -7 & 3 & v_2 - 3\,v_1 \\ 0 & 0 & 0 & v_3 - v_1 + v_2 \end{bmatrix}$$

Conclusion: **this system is not consistent for every b**

It is easy to find v1, v2, v3 such that v3-v1+v2 is nonzero

```
>
```