# Properties of Matrix Operations

$A$, $B$, $C$ are matrices of appropriate sizes.

$r$ and $s$ are (arbitrary) scalars.

$I_m$ and $I_n$ are $m \times m$ and $n \times n$ identity matrices.

**Sums and Scalar Multiples**

**a.** $A + B = B + A$

**b.** $(A + B) + C = A + (B + C)$

**c.** $A + 0 = A$

**d.** $r(A + B) = rA + rB$

**e.** $(r + s)A = rA + sA$

**f.** $r(sA) = (rs)A$

**Matrix Multiplication**

**I.** If $B$ has columns $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_p$, <u>**then**</u> $AB = A[\mathbf{b}_1\ \mathbf{b}_2\ \cdots\ \mathbf{b}_p] = [A\mathbf{b}_1\ A\mathbf{b}_2\ \cdots\ A\mathbf{b}_p]$.

**II.** If $A = (a_{ij})$ and $B = (b_{ij})$, <u>**then**</u> $(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$.

**a.** $A(BC) = (AB)C$

**b.** $A(B + C) = AB + BC$

**c.** $(A + B)C = AC + BC$

**d.** $r(AB) = (rA)B = A(rB)$

**e.** $I_m A = A = AI_n$

**Transpose and other Operations**

**a.** $(A^T)^T = A$

**b.** $(A + B)^T = A^T + B^T$

**c.** $(rA)^T = rA^T$

**d.** $(AB)^T = B^T A^T$

**Can be true but in general <u>NOT</u>**

**a.** $AB \neq BA$

**b.** $AB = AC \not\Rightarrow B = C$

**c.** $AB = 0 \not\Rightarrow A = 0$ or $B = 0$