Math 22 Fall 2003
First Hour Exam

1. (30) Consider the linear system

$$\begin{aligned} 2x_1 + 2x_2 \qquad\quad + 4x_4 &= 0 \\ x_1 + 2x_2 + 2x_3 + 5x_4 &= 2 \\ 3x_1 + 4x_2 + 2x_3 + 9x_4 &= 1 \\ 3x_1 + 6x_2 + 6x_3 + 15x_4 &= 0. \end{aligned}$$

(i) Write down the matrix of coefficients A and the augmented matrix of this system.

(ii) Put the augmented matrix of (i) into echelon form. State which operations are used to do this.

(iii) State if the given system has no solution, a unique solution or infinitely many solutions. Justify your answer.

(iv) Use your answer in (ii) to determine a spanning set for the set of all solutions to the associated homogeneous system $A\mathbf{x} = \mathbf{0}$.

2. (25) Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^4$ defined by

$$T(x_1, x_2, x_3) = (x_1 + 2x_2, 4x_1 - x_2 + 3x_3, 3x_2 + 5x_3, x_1 + x_3)$$

(i) What is the matrix of T?

(ii) Is T onto? Justify your answer.

(iii) Is T one-one? Justify your answer.

(iv) Are the columns of T linearly independent? Give a reason for your answer.

3. (15) Given

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{pmatrix}.$$

Show that A is invertible and find $\det(A^{-1})$.

4. (30) Complete each of the following statements.

(1) [4] A system of m linear equations in n unknowns which is given by the matrix equation $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbf{R}^m$ if and only if

(2) [4] The columns of a matrix A are linearly independent if and only if .

Do not give the definition of linearly independent columns.

(3) [3] A linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ is one-one if

(4) [3] If $T : \mathbf{R}^3 \to \mathbf{R}^3$ is the linear transformation defined by $T(\mathbf{x}) = a\mathbf{x}$ for some scalar a and all $\mathbf{x} \in \mathbf{R}^3$, then the matrix of T is

(5) [3] An $n \times n$ matrix A is invertible if

Give the definition of invertible.

(6) [2] For a matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

the ij-entry of A^T is

(7) [3] An elementary matrix is obtained by

(8) [4] If A is an $m \times n$ matrix and B is an $n \times p$ matrix whose ith column is $\mathbf{b}_i$, then the ith column of AB is

(9) [4] Every set of n vectors in $\mathbf{R}^m$ is linearly dependent provided

Do not give the definition of linearly dependent.