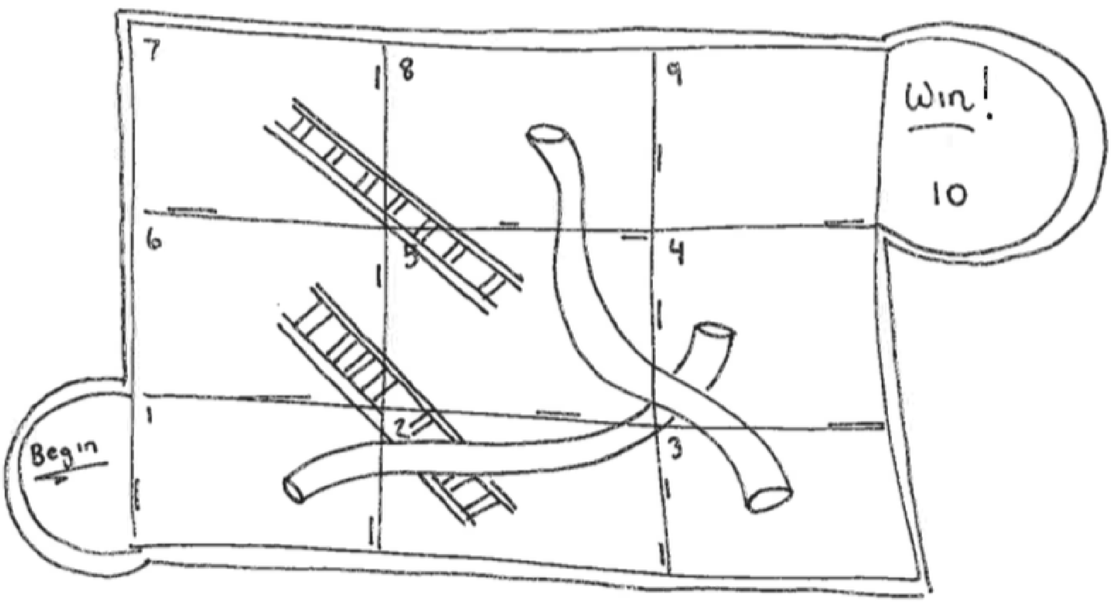
Games can be thought of as Markov Chains! In this assignment, we'll work with a simplified version of the game Chutes and Ladders.

Fun fact: A group of people at the University of Chicago built the Markov chain for the full version of Chutes and Ladders and found that the expected length of the game is approximately 40 moves. We'll do the same sort of analysis with a simplified version of the game.



Rules: Each player starts with a token on the starting square 0 and will move the token by the number of squares indicated by a spinner with three equal regions, labeled 1, 2, and 3. (This is in place of the roll of a fair die in the original game.) If, after advancing a few steps according to the value of the spinner, a player's token lands at the bottom of a ladder, the player must immediately move the token up to the end of the ladder. If a player lands on the top of a chute, then player must immediately slide the token down to the end of the chute. Climbing a ladder or sliding down a chute does not count as a separate turn. In this variation, we will assume that a player must spin the exact number to reach the final square to win. *Eg.* If you are on square 9, you must spin the value 1 in order to advance to the final square, otherwise you remain at square 9. Since each player in this game is moving independently of the others, we will only focus on the trajectory of a single person.

Problems: The first two problems can be solved without using the computer, enjoy the sunshine while you puzzle through them.

1. Write down a directed graph with the associated transition probabilities for this game. *Hint*: Your transition matrix should be a 7×7 matrix, including the "begin" and "end" states.

2. By looking at the board itself, what is the fewest number of turns it could take to win? By counting possible paths through this game, what is the probability that you win in exactly this many turns? Use the Markov chain to verify your answers.

3. What is the probability that you win in five or fewer turns? What is the probability that you win in eight or fewer turns? What is the probability that it takes *exactly* five turns to win? What is the probability that it takes *exactly* eight turns to win?

4. Let X be the random variable that is the number of turns until you win. Use the tail-sum formula for the expectation to find an estimate for the number of turns it will take to win. The tail-sum formula is given by:

$$E(X) = \sum_{k=0}^{\infty} P(X > k).$$

   Hint: You do not need to find the value of an infinite sum, use the computer to find, say, the first twenty values of the sum - your estimate will certainly be close enough.