Math 20
Homework 7
Due: August 14, 2015

Solve the following problems and explain your reasoning.

Book problems: 8.1.7, 8.1.17, 9.1.8, 9.1.16, 9.3.14

6. The following theorem is a more general version of the central limit theorem:
Lindeberg's Theorem: Let $X_1, X_2, ...$ be a sequence of independent random variables. Set $\mu_k = E(X_k)$ and $\sigma_k^2 = V(X_k)$. Let $S_n = X_1 + ... + X_n$. Then S_n has mean $m_n = \mu_1 + ... + \mu_n$ and variance $s_n^2 = \sigma_1^2 + ... + \sigma_n^2$. For a fixed $\epsilon > 0$, define the *truncated random variables*:

$$U_k = \begin{cases} X_k - \mu_k & \text{if } |X_k - \mu_k| \leq \epsilon s_n \\ 0 & \text{if } |X_k - \mu_k| > \epsilon s_n \end{cases}.$$

If $s_n \to \infty$ and for every $\epsilon > 0$ we have:

$$\frac{1}{s_n^2} \sum_{k=1}^{n} E(U_k^2) \to 1,$$

then $X_1, X_2, ...$ satisfies the conclusion of the Central Limit Theorem.

Using Lindeberg's Theorem, show that if $X_k =$ the number of inversions induced by k in a permutation of $1, 2, 3, ..., n$, then $\{X_i\}_{i=1}^{\infty}$ satisfies the conclusion of the Central Limit Theorem.

7. Suppose that a fair die is rolled 100 times. Let X_i be the value obtained on the ith roll. Compute an approximation for:

$$P\left(X_1 X_2 \ldots X_{100} \leq a^{100}\right),$$

for $1 < a < 6$.