Math 20
Homework 6
Due: August 7, 2015

Solve the following problems and explain your reasoning.

Book problems: 7.1.2, 7.1.10, 7.2.6, 8.1.5, 8.1.10, 8.2.8

7. Suppose X and Y are two random variables with density functions given by the *deflated tent function*:

$$f_X(x) = \begin{cases} 0 & \text{if } |x| > 1 \\ \frac{3}{2}(x+1)^2 & \text{if } -1 \leq x \leq 0 \\ \frac{3}{2}(x-1)^2 & \text{if } 0 < x \leq 1 \end{cases},$$

and the *flattened box*:

$$f_Y(x) = \begin{cases} 1/2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases},$$

respectively. Use the convolution to compute the density function for $X + Y$.

8. Let $p(x)$ and $q(x)$ be two polynomials of degree m and n respectively. We can think of these as vectors by using the coefficients. For instance, if $p(x) = p_0 + p_1 x + p_2 x^2 + ... + p_m x^m$, then we can uniquely encode $p(x)$ as a vector using $\hat{p} := (p_0, p_1, ..., p_m)$. Using the (discrete) convolution, write down a formula for the product of $p(x)$ and $q(x)$ in terms of $\hat{p}$ and $\hat{q}$ (you can use the usual convention of writing the convolution as an infinite sum with the understanding that $\hat{p}$ and $\hat{q}$ are zero for indices outside the ranges of 0 and m, and 0 and n, respectively).

[Hint for problem 7.1.10: Use problem 8 to get the convolution for the distributions of a and b in terms of coefficients of a certain polynomial. Then you can use (without proof) the fact that if $n = pq$, with p the smallest prime divisor of n, then:

$$\frac{1}{n}\left(1 + x + x^2 + ... + x^{n-1}\right) = \frac{1}{p}(1 + x + x^2 + ... + x^{p-1})\frac{1}{q}f(x),$$

where $f(x)$ is some polynomial. You might also find it useful to look at the solution to problem 7.1.9.]