1.2.1 ALL 8 SUBSETS OF Ω ARE ENUMERATED AS FOLLOWS:

$$S_1 = \emptyset \qquad S_3 = \{b\} \qquad S_5 = \{a,b\} \qquad S_7 = \{b,c\}$$
$$S_2 = \{a\} \qquad S_4 = \{c\} \qquad S_6 = \{a,c\} \qquad S_8 = \{a,b,c\}$$

USING DEFINITION 1.2 IN THE TEXT:

$$P(S_1) = 0, \quad P(S_2) = m(a) = \frac{1}{2}, \quad P(S_3) = m(b) = \frac{1}{3},$$
$$P(S_4) = m(c) = \frac{1}{6}, \quad P(S_5) = \frac{5}{6}, \quad P(S_6) = \frac{2}{3}, \quad P(S_7) = \frac{1}{2}, \quad P(S_8) = 1$$

1.2.6 $\Omega = \{1, \ldots, 6\}$ AND WE KNOW $m(i) \sim i$. SINCE

$$P(\Omega) = 1 = \sum_{i=1}^{6} m(i) = \sum_{i=1}^{6} c \cdot i$$

CONSTANT OF PROPORTIONALITY

$$= c \sum_{i=1}^{6} i = c \cdot 75$$

WE MUST HAVE $c = \frac{1}{75}$. WE NEED TO COMPUTE $P(\{2,4,6\})$.

$$P(\{2,4,6\}) = m(2) + m(4) + m(6)$$
$$= \frac{1}{75}\left[2 + 4 + 6\right] = \frac{12}{75}$$

1.2.18 (b) $1 \geq P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Updownarrow$$

$$P(A \cap B) + 1 \geq P(A) + P(B)$$

$$\Updownarrow$$

$$P(A \cap B) \geq P(A) + P(B) - 1$$

1.2.19 SET $D = A \cup B$. THEN BY THM 1.4 WE HAVE:

$$P(A \cup B \cup C) = P(D \cup C) = P(D) + P(C) - P(D \cap C)$$

$$= P(A \cup B) + P(C) - P((A \cup B) \cap C)$$

USED THM 1.4 AGAIN & THE FACT THAT $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$$= P(A) + P(B) - P(A \cap B) - P(A \cap C \cup B \cap C) + P(C)$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

1.2.22 $\Omega = \{1, 2, 3, \ldots\}$

GIVEN $n \in \Omega$, WE DEFINE: $m(n) = (5/6)^{n-1} (1/6)$

THIS IS A DISTRIBUTION FUNCTION SINCE $m(n) \geq 0$ FOR EVERY n AND

$$\sum_{n=1}^{\infty} m(n) = \sum_{n=1}^{\infty} (5/6)^{n-1} (1/6)$$ THIS IS A GEOMETRIC SERIES

$$= \frac{1}{6} \frac{1}{1 - 5/6} = \frac{1}{6} \cdot \frac{6}{1} = 1.$$

SO $m : \Omega \to \mathbb{R}$ IS A DISTRIBUTION.

1.2.28 (a) ROUGHLY $\frac{1}{3}$ OF ALL POSITIVE INTEGERS SHOULD BE A MULTIPLE OF 3.

(b) $P_3(N) = \dfrac{\#\text{ OF INTEGERS BETWEEN } 1 \,\&\, N \text{ DIVISIBLE BY } 3}{N}$. AND # OF INTEGERS BETWEEN 1 & N DIVISIBLE BY 3 IS $\lfloor \frac{N}{3} \rfloor$, HERE $\lfloor \cdot \rfloor$ IS THE FLOOR FUNCTION.

NOW WE WANT TO KNOW $\lim_{N \to \infty} \dfrac{\lfloor \frac{N}{3} \rfloor}{N}$. WE HAVE THE FOLLOWING BOUNDS:

$$\frac{N}{3} - 1 \leq \left\lfloor \frac{N}{3} \right\rfloor \leq \frac{N}{3}.$$

SO SINCE $\lim_{N \to \infty} \dfrac{\frac{N}{3} - 1}{N} = \dfrac{1}{3} = \lim_{N \to \infty} \dfrac{\frac{N}{3}}{N}$, BY THE SQUEEZE THM:

$$\lim_{N \to \infty} P_3(N) = \frac{1}{3}.$$

(c) IF A IS A FINITE SET, THEN LET $\#(A) = \#$ OF ELEMENTS IN A $= M$.

THEN $P(A) = \lim_{N \to \infty} \dfrac{A(N)}{N} \leq \lim_{N \to \infty} \dfrac{M}{N} = 0$.

IF $A = \mathbb{Z}_{>0}$ THEN:

$$P(\mathbb{Z}_{>0}) = \lim_{N \to \infty} \frac{\mathbb{Z}_{>0}(N)}{N} = \lim_{N \to \infty} 1 = 1.$$

2.2.2

(a) $1 = \int_2^{10} Cx \, dx = \frac{Cx^2}{2}\Big|_2^{10} = C[50-2]$

so $C = \frac{1}{48}$

(b) $P(E) = \frac{1}{48}\int_a^b x \, dx = -\frac{a^2}{96} + \frac{b^2}{96}$

(c) $P(X>5) = \frac{1}{48}\int_5^{10} x \, dx = \frac{25}{32}$

$P(X<7) = \frac{1}{48}\int_2^{7} x \, dx = \frac{25}{32}$

$x^2-12x+35>0$, THE ROOTS ARE $x=5$ OR 7

AND $x^2-12x+35$ IS >0 WHEN $X<5$ AND AND $X>7$

THUS

$P(x^2-12x+35>0) =$

$P(x<5) + P(x>7)$

$= \frac{1}{48}\int_2^5 x \, dx + \frac{1}{48}\int_7^{10} x \, dx$

$= \frac{3}{4}$