NAME: ______________________________

Math 20

Summer 2015

Exam II

Instructions:

1. Write your name *legibly* on this page.
2. There are eight problems, some of which have multiple parts. Do all of them.
3. Explain what you are doing, and show your work. You will be *graded on your work*, not just on your answer. Make it clear and legible so I can follow it.
4. It is okay to leave your answers unsimplified. That is, if your answer is the sum or product of 5 numbers, you do not need to add or multiply them. Answers left in terms of binomial coefficients or factorials are also acceptable. However, do not leave any infinite sums or products, or sums or products of a variable number of terms.
5. There are a few pages of scratch paper at the end of the exam. I *will not look* at these pages unless you write on a problem "Continued on page…"
6. This exam is closed book. You may not use notes, calculators, or any other external resource. It is a violation of the honor code to give or receive help on this exam.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	15	
6	10	
7	10	
Total	75	

1. (10 points.) Suppose you have an urn containing N red and M blue balls. Balls are selected randomly and replaced until a red ball is obtained.

   (a) What is the probability that n draws are needed (and no more)?

   (b) What is the expected number of draws until a red ball is obtained (here we do not count the last draw of a red ball as a draw)?

2. (10 points.) The county hospital is located at the center of a square whose sides are 3 miles wide. If an accident occurs within this square, then the hospital sends out and ambulance. The road network is rectangular, so the travel distance from the hospital, whose coordinates are $(0, 0)$, to the point (x, y) is $|x| + |y|$. If an accident occurs at a point that is uniformly distributed in the square, find the expected travel distance of the ambulance.

3. (10 points.) Derive the *One-sided Chebyshev* inequality which asserts: If X is a random variable with $E(X) = 0$ and variance $V(X) = \sigma^2$, then for any $a > 0$:

$$P(X \geq a) \leq \frac{\sigma^2}{\sigma^2 + a^2}.$$

4. (10 points.)

(a) Let X be a random variable with mean 0 and variance 1. Find the smallest number n so that you can guarantee that $P(|X| \geq n) \leq 1/25$.

(b) Suppose that it is known that the number of cars produced in a Ford factory during a week is a random variable with mean 50. What can you say about the probability that a given week's production will exceed 75 cars? If the variance of a week's production is 25, what can you say about the probability that the production will be between 40 and 60 cars?

5. (15 points.) Suppose that X is a continuous random variable with density function:

$$f_X(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{3} & \text{if } 0 \leq t \leq 3 \\ 0 & \text{if } t > 3 \end{cases},$$

and Y is a continuous random variable with density function:

$$f_Y(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{2} & \text{if } 0 \leq t \leq 2 \\ 0 & \text{if } t > 2 \end{cases}.$$

(a) Find the density function for the random variable $X + Y$ (assuming the X and Y are independent).

(b) Use this to compute the probability that $\frac{1}{2} \leq X + Y \leq \frac{3}{2}$.

6. (10 points.) Suppose we have a Bernoulli trials process with probability of success equal to p. Let X be the number of trials until the first success. Derive the formula for $V(X)$.

7. (10 points.) We define the *moment-generating function* for a random variable X, denoted by $M_X(t)$, as:

$$M_X(t) = E\left(e^{tX}\right).$$

   (a) Find the moment-generating function for X when X is a binomial random variable with n trials and probability for success p and failure q. Your answer should not include *sums* or $\cdots$.

   (b) Use your answer from part (a) to find $E(X^3)$. [Hint: How is $\frac{d}{dt}M_X(t)$ related to $E(X)$?]

Extra paper for scratch work. We will not look at this page unless you write a note on a problem saying "Continued on page…"

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