Homework 8 – Due August 22, 2012

Be sure to write your first and last name on your homework. Please write neatly and staple all pages together. You should show all your work!

1. Consider the 'telephone' example we saw in class and on the last homework with transition matrix:
$$P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}.$$
   (a) Under what conditions is P absorbing?
   (b) Under what conditions is P ergodic but not regular?
   (c) Under what conditions is P regular?

   Be sure to justify your answers.

2. Find the fixed probability vector (the stable distribution) w for the matrices in the previous question that are ergodic.

3. [Section 11.3, Problem 24] A certain experiment is believed to be described by a two-state Markov chain with the transition matrix P, where
$$P = \begin{pmatrix} .5 & .5 \\ p & 1-p \end{pmatrix}$$
   and the parameter p is unknown. When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two approximately 80 percent of the time. Compute a sensible estimate for the unknown parameter p and explain how you found it.

4. Give a simple example of a Markov chain that is neither absorbing nor ergodic. Write down a graph that represents this Markov chain and a transition matrix for this Markov chain. Justify your answer.

5. If P is the transition matrix of an absorbing chain in canonical form, then what does $\lim_{n\to\infty} P^n$ look like?

6. Do problem number 23 from section 11.2, which describes a classic example from probability theory called *gambler's ruin*. Your answer should include both an explanation for why this scenario can be modeled by an absorbing Markov chain, what the general transition matrix, and an estimate for w_{50}.

7. Do problem number 24 from section 11.2. (The part that says "Show that these conditions determine w_x" means that you should show that these conditions force the w_x to be unique solutions. This may be challenging. It may help to think of w as a column vector with w_x the components of the vector. What is Pw? What is $P^n w$? What is $\lim_{n\to\infty} P^n$? These questions are hints to lead you in the right direction.)