Homework 6 – Due August 8, 2012

Be sure to write your first and last name on your homework. You should show all your work!

1. Let S_{100} be the number of heads that turn up in 200 tosses of a fair coin. Estimate:
   (a) $P(S_{200} = 100)$.
   (b) $P(S_{200} = 90)$.
   (c) $P(S_{200} = 80)$.

2. The bank accepts rolls of pennies and gives 50 cents credit without counting. A roll contains 49 pennies 30% of the times, 50 pennies 60% of the time, and 51 pennies 10% of the time. How many rolls does the bank need to collect to have a 99% chance of a net loss?

3. Suppose you have a random walker starts at 0 on the x-axis. This guy makes steps of size 2 with probability 0.1, size 1 with probability 0.3, size 0 with probability 0.3, or size -1 with probability 0.3.
   (a) What is the probability that after 100 steps, the man is not on the positive side of the axis (so his position is ≤ 0).
   (b) What is the probability that after 100 steps, the man is in a position between 0 and 20?

4. Use the Central Limit Theorem to prove the Law of Large Numbers.

5. There is a Matlab function called `erf`. If you type `erf(y)`, it returns:

$$\texttt{erf(y)} = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2}\,dx.$$

   It is known that

$$\texttt{erf(a/sqrt(2))/2} = \int_0^a \varphi(x)\,dx,$$

   which is the area under the curve (the standard normal density function) from 0 to a. Test out a few values and compare them with the chart in your book to convince yourself.

   There is another Matlab function called `erfinv`. This is the inverse function to `erf`. So if `erf(x) = y`, then `erfinv(y) = x`.

   (a) Suppose you were interested in the 90th confidence interval. How many standard deviations from the mean are the endpoints of this interval? Now, write this number of standard deviations from the mean in terms of the function `erfinv` for the $Z\%$ confidence interval, where $Z = 100 \cdot z$. Your answer should be in terms of z (a number between 0 and 1). Check that your answer works for the 90% confidence interval.
   (b) In a poll, where p is unknown and could be any number between 0 and 1, how many people must you plan to interview to guarantee that your 90% confidence interval to be accurate within 0.03 (so that the length of the interval is 0.06)? What if you want your 90% interval to be accurate within k (a number between 0 and 1). Your answer should be in terms of k.
   (c) Answer question 59 on probability online and upload your code. Parts (a) and (b) should give you some guidance/ideas.