# Homework 5 – Due August 1, 2012

Be sure to write your first and last name on your homework. Please write neatly and staple all pages together. You should show all your work! Note there are TWO pages to this homework.

1. Assume that, during each second, a Dartmouth switchboard receives one call with probability 0.01 and no calls with probability 0.99. Use the Poisson approximation to estimate the probability that the operator will miss at most one call if she takes a 5-minute coffee break.

2. A baker blends 600 raisins and 400 chocolate chips into a dough mix and, from this, makes 500 cookies.

   (a) Find the probability that a randomly picked cookie will have no raisins.

   (b) Find the probability that a randomly picked cookie will have exactly two chocolate chips.

   (c) Find the probability that a randomly chosen cookie will have at least two bits (raisins or chocolate chips) in it.

3. Prove that if $X$ is a positive-valued random variable, then $E(X^k) \geq E(X)^k$ for all $k \geq 1$.

4. IQ scores are approximately normally distributed with mean 100 and standard deviation 15.

   (a) A person is a genius if they have an IQ of 140 or higher. What is the probability that a randomly selected person is a genius?

   (b) Mensa accepts people who have IQ's in the top 2%. What IQ must you have to be accepted into Mensa?

   (c) What is the probability that a randomly selected person has an IQ of exactly 100? (Hint: IQ is a continuous random variable.)

5. (a) Use Chebyshev's inequality to find a lower bound for the probability that the proportion of heads is between 0.4 and 0.6 when you flip a coin 20 times. Do this again for when you flip a coin 100 times and when you flip a coin 1000 times. (So you should have three different answers.)

   (b) Use coinTosses.m from Homework 2 to simulate these scenarios 2000 times. What does the actual probability appear to be for each scenario (when you flip 20 times, 100 times, 1000 times)? Does the answer agree with part (a)? How good of an approximation are the inequalities from part (a)?

   (c) In part (b), why do we need to simulate this scenario 2000 times? Why is the answer we get from our Matlab code close to the actual probability of getting a proportion between 0.4 and 0.6? (Hint: Think of Bernoulli trials. How is each of these 2000 trials a Bernoulli trial?)

6. (a) Similarly to part (a) of the previous problem, use Chebyshev's inequality to find a lower bound for the probability that the proportion of clubs is between 0.2 and 0.3 when you draw a card from a deck 20 times. Do this again for when you draw from a deck 100 times and 1000 times.

(b) Similarly to part (b) of the previous problem, simulate these scenarios 2000 times. What does the actual probability appear to be for each scenario? Does this answer agree with part (a)? Note that for this problem, you don't have to start from scratch to write a Matlab program that simulates these scenarios. You should only need to make a few edits to coinTosses.m.

7. For this problem, you'll be editing the code cointossesLLN.m (found on website under Syllabus).

   (a) Edit this code so that it returns the average difference between the number of heads and the expected number of heads when you flip $n$ coins $num\_simulations$ number of times. So for example, after you've made your changes, cointossesLLN(10, 1000) should flip 10 coins, record the difference between the actual number of heads and the expected number of heads (here you should use "abs" for absolute value of the difference), repeat this experiment 1000 times and return the average (absolute value of the) difference between the number of heads and the expected number of heads. What is the average difference when $n = 10$? What about $n = 100$? $n = 1000$? You should use at least 1000 simulations for each.

   (b) Explain in words why it seems that the average difference seems to increase as $n$ increases. Why doesn't this contradict the law of large numbers?

**Extra Credit:** Do problem numbers 45 and/or 46 of the online homework.