Math 20 – Worksheet, Oct. 19

Name:

1. A card is drawn at random from a deck consisting of cards numbered 2 through 10. A player wins 1 dollar if the number on the card is odd and loses 1 dollar if the number is even. What is the expected value of his winnings?

2. A card is drawn at random from a deck of playing cards. If it is red, the player wins 1 dollar; if it is black, the player loses 2 dollars. Find the expected value of the game.

3. In Las Vagas, when a player bets on a number (0, 00, 1, 2, ..., 36), he receives 36 dollars if the ball stops on this number, for a net gain of 35 dollars; otherwise, he loses his dollar bet. Find the expected value for his winning.

4. In a second version of roulette in Las Vagas, a player bets on red or black. Half of the numbers from 1 to 36 are red, half are black. If a player bets a dollar on black, and if the ball stops on black number, he gets his dollar back and another dollar. If the ball stops on a red number or on 0 or 00 he loses his one dollar. Find the expected winnings for this bet.

Math 20 – Worksheet, Oct. 23

Name:

1. A box contains two gold coins and three silver coins. You are allowed to choose successively coins from the box at random. You win 1 dollar each time you draw a gold coin and lose 1 dollar otherwise. After a draw, the coin is not replaced. Show that, if you draw until you are ahead by 1 dollar or until there are no more gold coins, this is a favorable game.

2. A multiple choice exam is given. A problem has four possible answers, and exactly one answer is correct. A student is allowed to choose a subset of the four possible answers as her answer. If her chosen subset contains the correct answer, the student receives three points, but she loses one point for each wrong answer in her chosen subset. Show that if he just guesses a subset uniformly and randomly her expected score is zero.

3. An insurance company has 1000 policies on men of age 50. The company estimates that the probability that a man of age 50 dies within a year is .01. Estimate the number of claims that the company can expect from beneficiaries of these men within a year.

4. Suppose we have an urn containing c yellow balls and d green balls. We draw k balls, without replacement, from the urn. Find the expected number of yellow balls drawn.

Math 20 – Worksheet, Nov. 2

Name:

1. A fair coin is tossed 100 times. The expected number of heads is 50, and the standard deviation for the number of heads is $\sqrt{100 \cdot 1/2 \cdot 1/2} = 5$. What does Chebyshev's Inequality tell you about the probability that the number of heads that turn up deviate from the expected number 50 by three or more standard deviations (i.e., by at least 15)?

2. Let X be a random variable with $E(X) = 0$ and $V(X) = 1$. What integer value k will assure us that $P(|X| \geq k) \leq .01$?

3. Let S_n be the number of successes in n Bernoulli trials with probability p for success on each trial. Show, using Chebyshev's Inequality, that for any $\epsilon > 0$

$$P(|\frac{S_n}{n} - p| \geq \epsilon) \leq \frac{p(1-p)}{n\epsilon^2}.$$

4. Find the maximum value for $p(1-p)$ if $0 < p < 1$. Using this result and problem 3 , show that the estimate

$$P(|\frac{S_n}{n} - p| \geq \epsilon) \leq \frac{1}{4n\epsilon^2}$$

is valid for any p.

Math 20 – Worksheet, Oct. 28

Name:

1. A number is chosen at random from the set $S = \{-1, 0, 1\}$. Let X be the set number chosen. Find the expected value, variance, and standard deviation.

2. A random variable X has the distribution $P(0) = 1/3$, $P(1) = 1/3$, $P(2) = 1/6$, and $P(4) = 1/6$. Find the expected value, variance and standard deviation.

3. X is a random variable with $E(X) = 100$ and $V(X) = 15$. Find a) $E(X^2)$. b) $E(3X + 10)$. c) $E(-X)$. d) $V(-X)$. e) $D(-X)$.

Math 20 – Worksheet, Nov. 6

Name:

1. Let S_{100} be the number of heads that turn up in 100 tosses of a fair coin. Use the Central Limit Theorem to estimate (a) $P(S_{100} \leq 45)$; (b) $P(45 < S_{100} < 55)$; (c) $P(S_{100} > 63)$.

2. Let S_n be the number of successes in n Bernoulli trials with probability .8 for success on each trial. Let $A_n = S_n/n$ be the average number of successes. In each case give the value for the limit, and give a reason for your answer. (a) $\lim_{n\to\infty} P(An = .8)$; (b) $\lim_{n\to\infty} P(.7n < S_n < .9n)$; (c) $\lim_{n\to\infty} P(S_n < .8n + .8\sqrt{n})$