Math 20, Probability **Spring 2018** **S. Nanda**

Practice problems - X-hour May 2, 2018

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1. Cumulative Distribution

   Let X be a continuous random variable associated with some experiment and density function $f_X(x)$.
   $P(X \leq x)$ is called the *cumulative distribution function* and is denoted by $F_X(x)$.

   If the cumulative distribution function is $F_X(x)$ then the density function $f_X(x) = \frac{d}{dx}F_X(x)$.
   Find the density function for the following cumulative distribution function for a random variable X

$$F_X(x) = \begin{cases} 0 \text{ if } x < -1 \\ \frac{(x+1)}{2} \text{ if } -1 \leq x \leq 1 \\ 1 \text{ if } x > 1 \end{cases}$$

   Plot $F_X(x)$. Check that $f(x)$ that you found satisfies the definition of a density function.

2. The Norwich Beer Company runs a fleet of trucks along the 100 mile road from Hangtown to Dry Gulch. The trucks are old, and are apt to break down at any point along the road with equal probability.

   (a) Where should the company locate a garage so as to minimize the expected distance from a typical breakdown to the garage? In other words, if X is a random variable giving the location of the breakdown, measured, say, from Hangtown, and b gives the location of the garage, what choice of b minimizes $E[|X - b|]$? Assume X has uniform distribution on $[0, 100]$.

   (b) Now suppose X is not distributed uniformly over $[0, 100]$, but instead has density function $f_X(x) = 2x/10000$. Then what choice of b minimizes $E(|X - b|)$?

3. If X and Y are any two random variables, then the covariance of X and Y is defined by

$$cov[X, Y] := E[(X - E[X]))(Y - E[Y])].$$

   Note that $cov(X, X) = V(X)$. Show that, if X and Y are independent, then $cov[X, Y] = 0$; It does not follow that if $cov[X, Y] = 0$ then X and Y not independent. Think of an example where $cov[X, Y] = 0$ but X, Y are not independent.