

Try the problems marked \diamond in class. Work in groups.

1. Conditional Probability

- (a) \diamond A builder says that he will complete a building in 1 year at most. Suppose that the probability the builder will finish in less than y years is $\frac{y}{2}$, $0 \leq y \leq 1$. Given that builder is still working at it after 10 months what is the probability that the builder will take a full year to complete the building?
- (b) A deck of 52 cards is divided randomly into 4 equal piles. Compute the probability that each pile has exactly one ace. Use multiplication rule for conditional probabilities.

2. Joint Distributions

- (a) \diamond Suppose that 3 balls are picked at random from an urn containing 3 red, 4 white, and 5 blue balls. Let X and Y denote, respectively the number of red and white balls chosen. Write down the joint probability distribution function for X and Y . That is for each i, j find $m(i, j) = P(X = i, Y = j)$. Put it in tabular form

$i \downarrow$; $j \rightarrow$	0	1	2	3	Row sum = $P(\{X = i\})$
0					
1					
2					
3					
Column sum = $P(\{Y = j\})$					

What are the marginal

distributions of X and Y .

- (b) Suppose 15% of families in a community have 0 children, 20% have 1 child, 35% have 2 and 30% have 3 children. Suppose that in each family, each child is equally likely (independently) to be a boy or a girl. If a family is chosen at random from this community, then B - the number of boys and G - the number of girls in this family will have what joint probability distribution?

Find the marginal distributions.

3. Fixed Point Problem An incompetent assistant places 6 letters at random into envelopes without checking that the address on the envelope matches the addressee of the letter. What is the probability that no letter goes into the correct envelope.

4. Binomial Distribution

A drug is assumed to be effective with an unknown probability p . To estimate p the drug is given to n patients. It is found to be effective for m patients. The method of maximum likelihood estimate gives best estimate for p . By this method, we should choose the value for p that gives the highest probability of getting what we got on the experiment. Assume that the experiment is a Bernoulli trials process with probability p for success. What is the best estimate for p .