SET THEORY

1 BASICS ABOUT SETS

Probability theory uses the language of sets. As we have seen probability is defined and calculated for sets. This is a review of some basic concepts from set theory that are used in this class.

Definition: A set is a collection of some items (elements). We often use capital letters to denote a set.

To define a set we can list all the elements, or describe what the set contains in **curly brackets**. For example

- a set A consists of the two elements ♣ and ♡. We write $A = \{\clubsuit, \heartsuit\}$.
- a set of positive even integers can be written thus: $A = \{2, 4, 6, 8, \ldots\}$ where the $\ldots$ stand for all the subsequent even integers. We could also write $A = \{2k \mid k \in \mathcal{Z}^+\}$.

We always use **curly brackets** to denote the collection of elements in a set.

To say that a belongs to A, we write $a \in A$. To say that an element does not belong to a set, we use $\notin$. For example, we may write $1 \notin A$ if A is the set of even integers. Note that ordering does not matter, so the two sets $\{\clubsuit, \heartsuit\}$ and $\{\heartsuit, \clubsuit\}$ are equal.

Some important sets used in math are given below.

- The set of natural numbers, $\mathcal{N} = \{1, 2, 3, \ldots\}$.
- The set of integers, $\mathcal{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
- The set of rational numbers $\mathcal{Q}$.
- The set of real numbers $\mathcal{R}$.

We can also define a set by mathematically stating the properties satisfied by the elements in the set. In particular, we may write $A = \{x \mid x \text{ satisfies some property}\}$ or $A = \{x : x \text{ satisfies some property}\}$ The symbols $\mid$ and : are pronounced "such that."

- The set of complex numbers $\mathcal{C} = \{a + bi : a, b \in , \& \ i = \sqrt{-1}\}$
- Closed intervals on the real line. For example $A = \{x : x \in [2, 3]\}$ is the set of all real numbers x such that $2 \leq x \leq 3$.
- Open intervals on the real line. For example $A = \{x : x \in (1, 3)\}$ is the set of all real numbers x such that $1 < x < s$.
- Similarly, $[1, 2)$ is the set of all real numbers x such that $1 \leq x < 2$.

Notice in the last example no curly brackets are used but from the description it is clear what the set is.

2 VENN DIAGRAMS

Venn diagrams are very useful in visualizing sets and relations between them. In a Venn diagram any set is depicted by a closed region. Figure 1 shows an example of a Venn diagram. In this figure, the big rectangle shows the universal set or sample space Ω. The shaded area shows a set A. The figure on the right shows two sets A and B, where $B \subset A$. Here the sets are depicted only by their boundaries.

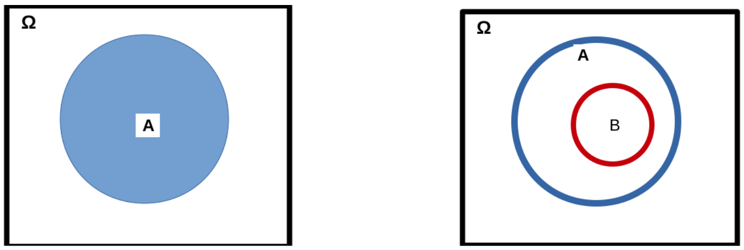

Figure 1: Venn Diagram for a set A (left) and $B \subset A$

3 SET OPERATIONS

The **union** of two sets is a set containing all elements that are in A or in B (possibly both). For example,$\{1,2\} \cup \{2,3\} = \{1,2,3\}$. Thus, we can write $x \in (A \cup B) \iff (x \in A)$ or $(x \in B)$. Note that $A \cup B = B \cup A$. We can write the union of many sets- say n sets, $A_1, A_2 \ldots, A_n$ as $A_1 \cup A_2 \cup \ldots \cup A_n$. This is a set containing all elements that are in at least one of the sets. We can write this union more compactly by $\cup_{i=1}^{n} A_i$. Viewed as events $\cup_{i=1}^{n} A_i$ means at least one of the n events occurs.

The **intersection** of two sets written $A \cap B$ contains all elements that are common to both A and to B. The intersection of n sets is written as $A_1 \cap A_2 \cap \ldots \cap A_n$ and compactly as $\cap_{i=1}^{n} A_i$. Viewed as events $\cap_{i=1}^{n} A_i$ means all n of the events occur. For example,$\{1,2\} \cap \{2,3\} = \{2\}$.

Two sets A and B are **mutually exclusive or disjoint** if they do not have any shared elements; i.e., their intersection is the empty set, $A \cap B = \emptyset$. More generally, several sets are called disjoint if they are pairwise disjoint, i.e., no two of them share a common elements.

The **complement** of a set A is denoted $\sim A$ or A^c or $\bar{A}$ and consists of all elements NOT in A i.e elements in the universal set Ω but are not in A. We can also say the complement of a set A in B. This set is all the elements in B that are not in A. This is the complement of A relative to the set B. For example,A^c in $B = \{1\}$ in previous example.

Figure 2 shows various set operations. Instead of Ω the sample space is called $\mathcal{U}$. The shaded area is the set (or event) of interest. For example $A \cup B'$ is represented in row 3 and column 3 in Figure 2.

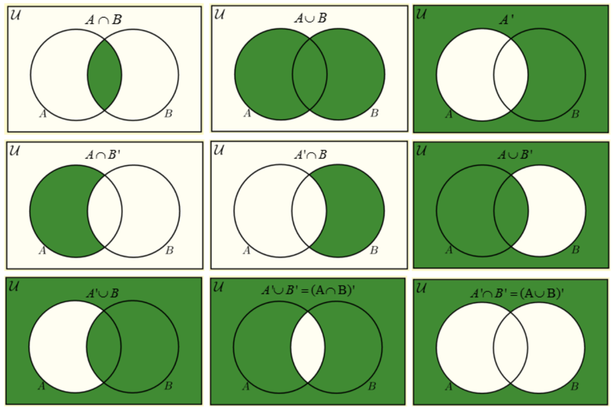

Figure 2: thanks online math learning

1. Use English phrases to describe the Venn diagrams in Figure 2.

2. Draw Venn diagrams corresponding to the foll situation. 28 students were surveyed to see if they ever had dogs or cats for pets at home.
   (a) 7 students said they had only ever had a dog.
   (b) 6 students said they had only ever had a cat.
   (c) 10 students said they had a dog and a cat.
   How many students had no pets?

3. Draw a Venn diagram for three disjoint sets A, B, C.

4. Draw a Venn Diagram which divides the twelve months of the year into the following two groups: Months whose name begins with the letter J and months whose name ends in *ber*.

5. 38 students were surveyed to see if they ever had dogs or cats or birds for pets at home.
   (a) 6 students said they had only ever had a dog.
   (b) 6 students said they had only ever had a cat.

(c) 10 students said they had a dog and a cat.

(d) 4 students said they had only ever had a bird.

(e) 3 students said that they had had a cat and a bird

How many students had no pets. How many had dogs. How many had all three?

6. Write the following in an alternative form using $\cup, \cap$ and complement of a set.

(a) $(A \cup B \cup C)^c$

(b) $(A \cap B \cap C)^c$

(c) $A \sim (B \sim C)$

(d) $E \cap (\cup_{i=1}^n A_i)$. Draw a Venn diagram for this if $\cap_{i=1}^n A_i = \emptyset$.