1. Consider flipping of two fair coins. Let

$$X = \begin{cases} 1 & \text{if first coin comes up heads} \\ 0 & \text{if first coin is tails} \end{cases}$$

Let

$$Y = \begin{cases} 1 & \text{if second coin comes up heads} \\ 2 & \text{if second coin comes up tails} \end{cases}$$

Let $Z = X + Y$. Find the probability distribution of $Z$. Find the $E(Z)$.

2. Let $S_n$ be the number of heads in a fair coin toss. What is the limit as $n \to \infty$ of each of the following probabilities? Justify your answers.

   (a) (a) $P(n/2 - 100 < S_n < n/2 + 100)$

   (b) (b) $P(0.4n < S_n < 0.6n)$

   (c) (c) $P(S_n < 0.5n + 0.5\sqrt{n})$

3. Give an example of a Markov Chain that is neither absorbing nor ergodic.

4. A process moves on the integers $1, 2, 3, 4$ and 5. It starts at 1 and, on each successive step, moves to an integer greater than its present position, moving with equal probability to each of the remaining larger integers. State 5 is an absorbing state. Find the expected number of steps to reach state 5. This is problem 9, 11.2

5. Problems 13, 15, ~~19~~ section 11.2

6. Problems ~~9~~, 10, 11, 16 section 11.3

7. A Restaurant feeds 300 customers. On the average 20% of the customers order steak.

   (a) Give a range for the number of steaks ordered on any given day so that you can be 95% percent sure that the actual number will fall in this range.

   (b) How many customers should the Restaurant have, on the avergae to be atleast 95% sure that the number of customers ordering steak on that day falls in the 19% to 21% range.

8. Try to prove the weak law of large numbers using the central limit theorem.

9. TRUE Or FALSE?

   (a) The Central Limit Theorem (CLT) says that when you multiply a large number of random variables, the result will be a normal random variable.

   (b) One of the assumptions of a Poisson distribution is that the system has no memory

   (c) The WLLN gives a limiting value of the sum of independent, identically distributed random variables

(d) The CLT says that when you add a large number of random variables, the result will be a normal random variable.

(e) Chebyshev's inequality follows from the WLLN

(f) Chebyshev's inequality assumes that the random variable in questions has a symmetric distribution.

Additionally please look over the practice problems of mid-term 1 and 2 - in particular the true/False.