MATH 20: DISCRETE PROBABILITY
SPRING 2017
PRACTICE PROBLEMS MIDTERM II

Problem 1. TRUE OR FALSE

(1) Let $X_i, i = 1, \ldots, n$ be random variables with some distribution. Let $S_n = \sum_{i=1}^n X_i$. Then $\frac{S_n}{n}$ is a random variable.
(2) Let $X_i, i = 1, \ldots, n$ be independent random variables with some distribution, with finite expectation μ and finite variance σ. Let $S_n = \sum_{i=1}^n X_i$. The weak law of large numbers says that $\lim_{n\to\infty} \frac{S_n}{n} = \mu$.
(3) The weak law of large numbers implies that no matter what happens in the short run in the long run there will be a very high probability that the average of observations will be close to the expected value.
(4) A random variable X counts the number of successes in n Bernoulli trials, where the trials are done without replacement. Then X has a hypergeometric distribution.
(5) A random variable which has mean and variance λ has a Poisson distribution.

Problem 2. There are 5 yellow and 4 blue balls in an urn. 3 balls are drawn at random. What are the expected number of yellow balls drawn. If the first ball drawn is blue, find the expected number of yellow balls drawn.

Problem 3. Let X be the first time that a failure occurs in an infinite series of Bernoulli trials with probability p for success. What is $E(X)$? What is the expected number of tosses of a coin required to obtain the first tail?

Problem 4.
A number is chosen at random from the integers $1, 2, \ldots, n$. Let X be the number chosed. What is $V(X)$?

Problem 5.
Asume that the probability that there is a significant accident in a nuclear power plant during one year is 0.001. If there are 50 nuclear power plants in a country what is the probability of at least one accident during a year?

Problem 6. Let X and Y be two random variables defined on a finite sample space Ω. Assume that $X, Y, X+Y, X-Y$ all have the same distribution. Show that $P(X = 0) = 1$. What can you say about Y?

Problem 7. A communication system consists of n components, each of which functions independently with probability p. The total system will be able to operate effectively if atleast one-half of its components function. For what values of p is a 5-component system more likely to operate effectively that a 3 component system?
hint: write down the probability of each system being effective and compare them.

Problem 8. David is working on a math problem that he recognizes as having 3 possible approaches to the solution. The first approach will take him 2 hour to solve the problem. The second approach will take him 1 hour but he will be back to square one. The third approach will take him 1/2 hours with no results. Assume David is equally likely to choose any of the approaches, what is the expected time for him to solve the problem?

Problem 9. Assume X takes integer values and is uniformly distributed over $[0, 10]$. Give an upper bound for $P(|X - E(X)| > 4)$.

Problem 10. The number of TVs sold weekly at a store is a random variable with expected value 10. Give an upper bound on the probability that next week's sales exceed 18. Let the variance of number of TVs sold per week be 5. Give a lower bound for the probability that next week's sales will be between 8 and 12.

Problem 11. Let X be a random variable with $E(X) = \mu$ and $V(X) = \sigma^2$. Show that for every positive integer k

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$