MATH 20: DISCRETE PROBABILITY
SPRING 2017
PRACTICE PROBLEMS MIDTERM I

Problem 1. Mark True or False: No justification needed

(1) Let $\Omega = \{\omega_1, \omega_2, \ldots \omega_n\}$ be a sample space. Then it must be the case that $P(\Omega) = 1$

(2) The number of questions that you answer correctly on this review is an example of a discrete random variable.

(3) The expected value for a random variable must be a possible value of that random variable.

(4) The probability of a student randomly guessing answers to a true/false exam is best modeled with a binomial distribution.

(5) The formula for the binomial probability distribution takes into account both the probability of success as well as the probability of failure.

(6) A probability distribution for a discrete variable depicts all possible mutually exclusive events, with the sum of the corresponding probabilities equalling 1.0.

Problem 2. A coin comes up tails with probability p on any particular flip. Let the random variable X be the number of flips until, and including, the first time it comes up tails.

- What is the probability distribution of X?
- What is E(X)?

Solution. (This was done in class last on Wednesday - April 12, 2017)

Problem 3. There are 21 balls in an urn, 6 of them are blue, 7 are red, and 8 are yellow. If you pick 5 balls from the urn at random, what is the probability that x of them will be blue, and y of them will be red, for any x, y?

Solution.

Problem 4.

- A sample space Ω conists of the ordered triples (i, j, k) where i, j, k are integers in $\{1, 2, 3\}$ and are either all different or all the same. List the members of Ω.
- Give the sample space Ω for a student chosen at random from a class of 10.

Solution.

Problem 5. What is Stirlings formula?

Solution.

Problem 6. It has been discovered that 20% of major league baseball players use steroids. A certain drug test gives a positive for steroid use 99% of the time if you are using steroids, and 2% of the time if you are clean.

- (a) What is the probability of a random player testing positive?
- (b) What is the probability of being a steroid user if you test positive?

Solution.

Problem 7. If $P(A) = \frac{1}{2}, P(B) = 1/4, P(C) = \frac{1}{8}.P(A \cup B) = \frac{3}{4}$ Find each of the following:

- $P(A \cap B^c)$
- $P(A^c \cap B^c)$
- A best estimate lower bound for $P(A \cup B \cup C)$ given the above information.

Solution.

Find the best estimate lower bound, given additionally $P(A \cap C) = \frac{1}{10}, P(B \cap C) = \frac{1}{12}$.

Problem 8. John rolls 2 six-sided dice, and if the sum of the dice is divisible by 3, he wins \$6 dollars. If the sum is not divisible by 3, he loses \$3 dollars. What is John's expected winnings from playing this game?

Solution.

Problem 9. Refer to the hat check problem discussed in class.
In the hat check problem, assume that N people check in their hats and these are handed back randomly. Let $X_j = 1$ if the jth person gets her hat back. Otherwise $X_j = 0$. Find $E(X_j)$. Are X_j and X_k independent?

Solution.

Problem 10. In a survey of the chewing gum tastes of a group of baseball players, it was found that:

- 22 liked juicy fruit
- 25 liked spearmint
- 39 like bubble gum
- 9 like both spearmint and juicy fruit
- 17 liked juicy fruit and bubble gum
- 20 liked spearmint and bubble gum
- 6 liked all three
- 4 liked none of these

How many baseball players were surveyed?