# Homework 7

Due May 14, 2014

Please make sure to explain your answers to each of the following questions. Remember: a correct numerical answer without explanation is worth no points! Write up your answers legibly and logically. All of the questions assigned are important exam practice (including the ones not to be turned in).

1. Many believe the daily change of price of a company's stock is a normally distributed random variable with mean 0 and variance $\sigma^2$. That is, if $Y_n$ is the price on the $n$th day, then $Y_n = Y_{n-1} + X_n$ where the $X_i$ are an independent trial process of normally distributed random variables with parameters $\mu = 0$ and $\sigma$. Assume today's price ($Y_0$) is 100 and $\sigma = 1$. What is the probability the stock's price will exceed 105 after 10 days? You may use the fact that the sum of independent normal random variables is a normal random variable.

2. A sample is taken to estimate $f$ the fraction of men in a population. Find a sample size such that the probability of a sampling error less than 0.005 will be 99% or greater.

3. We have a coin which is either fair or has a 55% chance of heads (presumably, these are equally likely). In order to decide whether or not the coin is fair, we flip the coin 1000 times.

   (a) We guess the coin is biased if 525 or more flips are heads. Otherwise, we guess it is fair. What is the probability we arrive at a false conclusion? (This can happen in two ways)

   (b) With this guessing scheme, what is the best value to use as our threshold?

4. We roll a fair (6-sided) die 100 times. Let $X_i$ be the value of the $i$th roll. For $1 < a < 6$, compute an approximation for

$$P(X_1 \cdot X_2 \cdot \ldots \cdot X_{100} \leq a^{100})$$

   (Hint: how can we turn multiplication into addition?)

5. A telephone company needs a separate line for each call being made from exchange $B$ to the main exchange $A$. If they have 2000 customers at exchange $B$, each of whom uses a line an average of 2 minutes an hour during the peak hour, how many lines do they need to have a probability less than 0.01 of all lines being occupied? Use both the normal and Poisson approximations to find this value. (Hint: Use the normal approximation first, then use the value you find to start guessing which value works for the Poisson distribution)

6. Prove the Law of Large Numbers using the Central Limit Theorem.

Problems **not** to turn in:

1. Section 9.1 Exercises 1,2,3

2. Section 9.1 Exercise 6

3. Section 9.1 Exercise 14

4. Section 9.2 Exercises 1, 6