MATH 20, SPRING 2011
HOMEWORK #6

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This assignment will be due on Wednesday, May 18 at 12:30 p.m. in the box outside 105 Kemeny. Look for the boxes labeled "Math 20, Spring 2011" and put your assignment in the left ("IN") box.

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) Run the Coupon Collector simulator I used in class[1] 10 times using the setup for a standard poker deck ("Playing Cards").
   (a) Record the outcomes of your 10 simulations.
   (b) Calculate the expected value for the outcome of your simulations.
   (c) Take the largest and smallest values in your list of outcomes (call them M and m). Estimate the probability that the outcome is at least as big as your largest outcome and the probability that the outcome is no bigger than your smallest outcome.
   (d) Calculate the average of your outcomes and then use either bound on the variance given in class to estimate the probability that an outcome's difference from the expected value is at least as big as the difference between the average of your outcomes and the expected value.

(2) Suppose that the average number of accidents at an intersection is 2 a day.
   (a) Use Markov's inequality to bound the probability that at least 5 accidents will occur tomorrow.
   (b) Assuming that the accidents follow a Poisson distribution, calculate the probability that at least 5 accidents will occur tomorrow and compare this value with the bound in part (a).

(3) (Section 8.1, #6) Let S_n be the number of successes in n Bernoulli trials with probability p for success on each trial. Show, using Chebyshev's Inequality, that for any $\epsilon > 0$

$$P\left(\left|\frac{S_n}{n} - p\right| \geq \epsilon\right) \leq \frac{p(1-p)}{n\epsilon^2}.$$

(4) Please read this question carefully! Suppose that Dartmouth wants to enroll no more than 1100 students in the Class of 2016, and the admissions office wants some information based on previous years' admissions data. Look at the website http://www.dartmouth.edu/admissions/facts/admissions.html. On it, you will find admissions statistics for your class year.

[1] http://www-stat.stanford.edu/~susan/surprise/Collector.html

- Use the "Total Admitted" and "Total Enrolled" data *for your class year* to estimate the probability p that an admitted student will accept Dartmouth's offer.

Now assume that the acceptances can be modeled by Bernoulli trials. Given the probability you just calculated and the total number of students admitted *in your year*, what is the probability that Dartmouth would have had too many acceptances in your year? (Keep in mind that the Dartmouth admissions office didn't have access to the p you calculated above when they sent out your letters.)

(5) (Section 9.1, #14) A restaurant feeds 400 customers per day. On the average, 20 percent of the customers order apple pie.
   (a) Give a range (called a 95% confidence interval) for the number of pieces of apple pie ordered on a given day such that you can be 95% sure that the actual number will fall in this range. (Assume that everyone who orders apple pie orders exactly one piece.)
   (b) How many customers must the restaurant have, on the average, to be at least 95% sure that the number of customers ordering apple pie on that day falls in the 19-21% range?

Suggested problems: Section 8.1: 4-5, 7-8, 10-11, 16; Section 9.1: 1-5, 12-13, 16-18