MATH 20, SPRING 2011
HOMEWORK #5

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This assignment will be due on Wednesday, May 11 at 12:30 p.m. in the box outside 105 Kemeny. Look for the boxes labeled "Math 20, Spring 2011" and put your assignment in the left ("IN") box.

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) Let X be the number of spades in a randomly chosen poker hand. Calculate $E(X)$ and $V(X)$.

(2) (Section 6.2, #22) Let X and Y be two random variables defined on the finite sample space Ω. Assume that X, Y, $X + Y$, and $X - Y$ all have the same distribution. Prove that $P(X = Y = 0) = 1$.

(3) (Section 5.1, #16) Assume that, during each second, a Dartmouth switchboard operator receives one call with probability .01 and no calls with probability .99. Use the Poisson approximation to estimate the probability that the operator will miss at most one call if she takes a 5-minute coffee break.

(4) (Section 5.1, #24) When John Kemeny was chair of the Mathematics Department, he received an average of ten letters a day. On a certain weekday he received no mail and wondered if it was a holiday. To decide this he computed the probability that, in ten years, he would have at least one day without any mail. He assumed that the number of letters he received on a given day has a Poisson distribution. What probability did he find? *Hint:* Apply the Poisson distribution twice: first, to find the probability that, in 3000 days, he will have at least 1 day without mail, assuming each year has about 300 days on which mail is delivered.

(5) (Section 5.1, #38) A manufactured lot of buggy whips has 20 items, of which 5 are defective. A random sample of 5 items is chosen to be inspected.
  (a) Find the probability that the sample contains exactly one defective item if the sampling is done with replacement.
  (b) Find the probability that the sample contains exactly one defective item if the sampling is done without replacement.

(6) (Section 5.1, #43) The students in a certain class were classified by hair color and eye color. The conventions used were: Brown and black hair were considered dark, and red and blonde hair were considered light; black and brown eyes were considered dark, and blue and green eyes were considered light. They collected the data shown in Table 5.6 (on p. 204 in the textbook). Are these traits independent?

Suggested problems: Section 6.2: 2-5, 7, 10, 12, 29; Section 5.1: 1, 2, 4, 7-9, 17-20, 25, 35, 42