

MATH 20, SPRING 2011
HOMEWORK #4

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This assignment will be due on Wednesday, May 4 at 12:30 p.m. in the box outside 105 Kemeny. Look for the boxes labeled “Math 20, Spring 2011” and put your assignment in the left (“IN”) box.

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

- (1) An art gallery is audited, and it is found that 25% of the pieces in the gallery are fakes. One frequent buyer from the gallery has occasional lapses of judgment. When she buys a piece of art, she is wrong about whether it is an original or a fake 15% of the time. If she buys a piece thinking that it is an original, what is the probability that she is wrong?
- (2) (Section 4.1, #18) A doctor assumes that a patient has one of three diseases d_1 , d_2 , or d_3 . Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has d_1 , 0.6 if he has disease d_2 , and 0.4 if he has disease d_3 . Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?
- (3) (Section 4.1, #56 and #57) Let A and B be two events with positive probabilities. We say that A *attracts* B if $P(B | A) > P(B)$ and *repels* B if $P(B | A) < P(B)$.
 - (a) Prove that A attracts B if and only if B attracts A .
 - (b) Prove that A neither attracts nor repels B if and only if A and B are independent.
- (4) A box contains 10 disks of radii $1, 2, \dots, 10$, respectively.
 - (a) What is the expected value of the area of a disk selected at random from this box?
 - (b) What is the expected value of the circumference of a disk selected at random from this box?
- (5) (Section 6.1, #8) A royal family has children until it has a boy or until it has three children, whichever comes first. Assume that each child is a boy with probability $\frac{1}{2}$. Find the expected number of boys in this royal family and the expected number of girls.
- (6) (Section 6.1, #19) A multiple choice exam is given. A problem has four possible answers, and exactly one answer is correct. The student is allowed to choose a subset of the four possible answers as his answer. If his chosen subset contains the correct answer, the student receives three points, but he loses one point for each wrong answer in his chosen subset. Show that if he just guesses a subset uniformly and randomly his expected score is zero.

- (7) (Section 6.1, #22) Recall that in Exercise 1.1.14, we considered a town with two hospitals. In the large hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. We were interested in guessing which hospital would have on the average the largest number of days with the property that more than 60 percent of the children born on that day are boys. For each hospital find the expected number of days in a year that have the property that more than 60 percent of the children born on that day were boys. (Don't worry about evaluating your answer. You can leave it in sigma notation.)

Suggested problems: Section 4.1: 19, 39, 43, 49, 53, 63; Section 6.1: 1-5, 13, 15, 17