MATH 20, SPRING 2011
HOMEWORK #3

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This assignment will be due on Wednesday, April 20 at 12:30 p.m. in the box outside 105 Kemeny. Look for the boxes labeled "Math 20, Spring 2011" and put your assignment in the left ("IN") box.

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) An illegal gambling den has eight rooms, each named after a different animal. Suppose that the owner wants to place 16 identical tables in the rooms.
  (a) Find the total number of ways of doing this.
  (b) Find the number of ways of doing this if Horse Room and Ox Room must each contain at least two of the tables.
  (c) Find the number of ways of doing this if Peacock Room must not have any tables.

(2) (Section 4.1, #7) A (fair) coin is tossed twice. Consider the following events:
  - A: Heads on the first toss.
  - B: Heads on the second toss.
  - C: The two tosses come out the same.

  (a) Show that A, B, and C are pairwise independent but not independent.
  (b) Show that C is not independent of $A \cap B$.

(3) (Section 4.1, #17) Prove that if A and B are independent and each has probability less than 1, then each of the following pairs of events are independent.
  (a) A and $\widetilde{B}$
  (b) $\widetilde{A}$ and $\widetilde{B}$

(4) (Section 4.1, #34) Four women, A, B, C, and D, check their hats, and the hats are returned in a random manner. Let Ω be the set of all possible permutations of A, B, C, and D. Let X_j=1 if the j^{th} woman gets her own hat back and 0 otherwise. What is the distribution of X_j? Are the X_is mutually independent?

Suggested problems: Section 4.1: 1-6, 9, 14, 21, 28, 32