

Homework 6: Due Wednesday, April 15

Problem 1: Recall that a *flush* in poker is a hand in which all five of your cards have the same suit. Suppose that you are playing a game of poker in which each 2 is a “wild card”. That is, you can take each 2 to represent any other card. For example, if you have three hearts, the 2 of spades, and the 2 of diamonds, then this would be considered a flush because we can pretend that the two 2’s are other hearts. What is the probability of getting dealt a flush in this game? How much more likely is this than the probability of getting dealt a flush when there are no wild cards?

Problem 2: Suppose that you have a colleague that you would really prefer to avoid sitting next to.

a. Suppose that you and the colleague are amongst n people assigned to one circular table at a wedding. If the seats around that table were assigned randomly, what is the probability that you end up sitting next to your colleague?

b. Suppose that you and the colleague are amongst n people assigned to sit on one side of a long table for a discussion at a conference. If the seats on that side of the table were assigned randomly, what is the probability that you end up sitting next to your colleague?

Problem 3: For any n and k , we have $k \cdot \binom{n}{k} = n \cdot \binom{n-1}{k-1}$. Show this in the following two ways.

a. Plug in our developed formula for $\binom{n}{k}$ and show that both sides are equal.

b. Argue that each side counts the number of ways of selecting a committee consisting of k people, including a distinguished president of the committee, from a group of n people.