Homework 17: Due Monday, June 1

Problem 1: Suppose that all stocks are rated in one of four categories: Excellent, Good, Fair, and Poor. Suppose that we have the following.

- If a stock is rated Excellent one week, then the following week it has a 70% chance of being rated Excellent and a 30% chance of being rated Good.
- If a stock is rated Good one week, then the following week it has a 10% chance of being rated Excellent, an 80% of being rated Good, and a 10% chance of being rated Fair.
- If a stock is rated Fair one week, then the following week it has a 20% chance of being rated Good, an 75% of being rated Fair, and a 5% chance of being rated Poor.
- If a stock is rated Poor one week, then the following week it has a 20% chance of being rated Fair and an 80% chance of being rated Poor.

Show how to interpret this scenario as a Markov Chain, show that the Markov Chain is regular, and calculate the stable vector $\vec{w}$ for it.

Problem 2: Consider the following simple game. There are two states of the game called "Low" and "High". You begin in the Low state, and at each stage of the game you roll a pair of fair dice.

- Suppose that you are currently in the Low state. If you roll a 2, 3, or 4, you lose. If you roll a 12, you win. If you roll a 5, 6, or 7, you stay in the Low state. Otherwise (8,9,10,11), you move to the High state.
- Suppose that you are currently in the High state. If you roll a 9, 10, 11, or 12, you win. If you roll a 2, you lose. If you roll a 7 or 8, you stay in the High state. Otherwise (3,4,5,6), you move to the Low state.

Show how to interpret this game as an absorbing Markov Chain. Calculate the matrices Q, N, and B. What is the expected number of turns in the game? In the long run, do you expect to win more games or lose more games?