1. You take a joke gift to a holiday party, as does everyone invited. The host mixes up all the gifts and passes them out at random to the guests. Assuming there are ten guests, what is the probability you get the gift you took? What is the probability that some person gets the gift he or she took?

2. A seminar room has n chairs. There are k students in the room for a seminar, where $k \leq n$. The seminar takes a tea break, and every student leaves the room. Assuming they choose seats randomly when they return, what is the probability that no student is sitting in the seat in which he or she sat previously?

3. Let X be the random variable that counts the number of times we have to role a die until we have seen all six numbers on top. Find the expected value and variance of X.

4. The solution to the test problem

   An airline sells 63 tickets for a flight that holds 60 passengers. The probability that any one ticket holder does not show up for the flight is $\frac{1}{20}$. What is the probability that there will be a seat available for every ticket holder that shows up?

   is:

   Let $X = \#$ of no shows. Then the probability that there will be a seat available for every ticket holder that shows up is

$$\begin{aligned} P(X \geq 3) &= 1 - P(X < 3) \\ &= 1 - (b(63, \frac{1}{20}, 0) + b(63, \frac{1}{20}, 1) + b(63, \frac{1}{20}, 2)) \\ &\approx 1 - .384 = .616, \end{aligned}$$

   using the program BinomialProbabilities.

   Use the Poisson distribution to approximate the probability that there will be a seat available for every ticket holder that shows up. Compare your answer to the one above.

5. A coin is tossed n times. Let S_n be the total number of heads that come up, and let $A_n = S_n/n$ be the average number of heads per toss.
   (a) What are $E(A_n)$ and $V(A_n)$?
   (b) Use Chebyshev's inequality to find a lower bound for the probability

$$P(|A_n - E(A_n)| < .1)$$

   for $n = 50$, 100, and 200.
   (c) Find the three probabilities in part (b) exactly. Use the program CLTIndTrials. Do these probabilities behave as the Law of Large Numbers predicts?

6. Assume that X is a random variable with $E(X) = \mu$ and $V(X) = \sigma^2$. Let Y be the random variable defined by $Y = aX + b$ where a and b are constants, with $a \neq 0$.
   (a) Compute $E(Y)$ and $V(Y)$ in terms of μ and σ.
   (b) Express Y^*, the standardized version of Y, in terms of X, μ and σ.
   (c) How are X^* and Y^* related?