MATH 20, WORKSHEET 8

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DUE FRIDAY NOVEMBER 10TH

(1) Let P be the transition matrix of a Markov chain. Show that the ijth of the matrix P^n gives the probability that the Markov chain, starting in state s_i, will be in state s_j after n steps. Prove this by induction. Explicitly,

  (a) show that this is true for $n = 1$. (this is just by definition)
  (b) assuming that this is true for a given k, show that it is also true for $k + 1$.

(2) Given that

$$\lim_{n \to +\infty} \sum_{k=0}^{n} Q^k = N,$$

where Q is the submatrix of the transient states.
Let Y be the random variable that gives you the number of steps until absorption, given that you started in state s_i.

  (a) Compute $P(Y \geq k)$.
  (b) Show that $E[Y] = \sum_{l=1}^{t} n_{il}$, i.e., the sum of the entries in ith row of N.
  Please, do not use that the ij-entry n_{ij} of the matrix N is the expected number of times the chain is in state s_j, given that it starts in state s_i.

Hint: Look into Problem 1 of Worksheet 4.

(3) Let b_{ij} be the probability that an absorbing chain will be absorbed in the absorbing state s_j if it starts in the transient state s_i. Let B be the matrix with entries b_{ij}. Show that B is a $t \times r$ matrix, and

$$B = NR,$$

where N is the fundamental matrix and R is as in the canonical form.
Hint: The proof in the book Theorem 11.6 is a rough sketch, you need to formalize it.