MATH 20, WORKSHEET 6

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Due Friday October 27th

(1) Let S_n be the number of successes in n Bernoulli trials with probability p for success on each trial. Show, using Chebyshev's Inequality, that for any $\epsilon > 0$

$$P\left(\left|\frac{S_n}{n} - p\right| \geq \epsilon\right) \leq \frac{1}{4n\epsilon^2}.$$

Hint: show that $p(1-p) \leq 1/4$

(2) We have two coins: one is a fair coin and the other is a coin that produces heads with probability 3/4. One of the two coins is picked at random, and this coin is tossed n times. Let S_n be the number of heads that turns up in these n tosses. Does the Law of Large Numbers allow us to predict the proportion of heads that will turn up in the long run? After we have observed a large number of tosses, can we tell which coin was chosen? How many tosses suffice to make us 95 percent sure?

(3) A fair coin is tossed a large number of times. Does the Law of Large Numbers assure us that, if n is large enough, with probability > .99 the number of heads that turn up will not deviate from $n/2$ by more than 100?

(4) Show that, if $X \geq 0$, then $P(X \geq a) \leq E(X)/a$

(5) If X is normally distributed, with mean μ and variance σ^2, find an upper bound for the following probabilities, using Chebyshev's Inequality.

(a) $P(|X - \mu| \geq \sigma)$.
(b) $P(|X - \mu| \geq 2\sigma)$.
(c) $P(|X - \mu| \geq 3\sigma)$.
(d) $P(|X - \mu| \geq 4\sigma)$.

Now find the exact value using the program the table in Appendix A of Grinstead and Snell's book, and compare.