MATH 20, WORKSHEET 2
PROBABILITY AND COMBINATORICS

EDGAR COSTA

Warm up problems for the first due problem

(1) Assume that we have a coin where the probability of getting heads after a toss is u. What is the probability of getting tails?

(2) Assume that we toss the coin k times. What is the probability that we do not observe heads?

(3) Assume that we toss the coin k times. What is the probability that we observe heads only once?

(4) Assume that we toss the coin k times. What is the probability that we observe heads l times?

(5) Assume that we toss the coin k times. What is the probability that we observe heads at least once? Try to find an expression not involving sums.

(6) Assume that you have n coins, and you distribute them in two urns, A and B, where urn A has k coins. If an urn is chosen at random, let's say that the urn A has probability p of being chosen, and all the coins in that urn are tossed, what is the probability of you observing heads at least once?

Due Friday September 22

(1) A small boy is lost coming down Mount Washington. The leader of the search team estimates that there is a probability p that he came down on the east side and a probability $1-p$ that he came down on the west side. He has n people in his search team who will search independently and, if the boy is on the side being searched, each member will find the boy with probability u.

(a) Determine how he should divide the n people into two groups to search the two sides of the mountain so that he will have the highest probability of finding the boy.

(b) How does this depend on u?

(2) Prove the following binomial identity

$$\binom{2n}{n} = \sum_{j=0}^{n} \binom{n}{j}^2.$$

Hint: Consider an urn with n red balls and n blue balls inside. Show that each side of the equation equals the number of ways to choose n balls from the urn.