Homework 6

1. If $E(X) = E(Y) = 75, V(X) = 10, V(Y) = 12$ and $\text{Cov}(X, Y) = -3$. Find an upper bound for the $\Pr(|X - Y| > 15)$.

2. Suppose that the number of units produced daily by factory A is a random variable with mean 20 and standard deviation 3 whereas the number of units produced daily by factory B is a random variable with mean 18 and standard deviation 6. Assuming these two random variables are independent, derive an upper bound for the probability that more units are produced today at factory B than at factory A.

3. The servicing of a machine requires two separate steps with the time needed for the first step being an exponential random variable with mean .2 hours and the second step being an independent exponential random variable with mean .3 hours. If a repair person has 20 machines to service, approximate the probability that all the work can be completed within 8 hours.

4. On each bet, a gambler loses $1 with probability .7, loses $2 with probability .2 and wins $10 with probability .1. Approximate the probability that the gambler will be losing after his first 100 bets.

5. In 10000 independent tosses of a coin, the coin landed heads 5500 times. Is it reasonable to assume that the coin is not fair? Explain.

6. You conduct a survey where you ask a random sample of 400 likely votes whether they prefer candidate A or candidate B. The result of the survey is that 52% say they will vote for A and 48% say they will vote for B. How confident are you that candidate A will win? Explain.

7. If X and Y are independent uniform random variables on the interval $(0, 1)$, show that $E(|X - Y|^t) = 2/(t + 1)(t + 2)$.

8. The county hospital is located at the center of a square whose sides are 3 miles wide. If an accident occurs within the square, then the hospital sends out an ambulance.

The road network is rectangular so the travel distance from the hospital (whose coordinates are $(0,0)$) to the point (x, y) is $|x| + |y|$. Assuming that an accident occurs at a point that is uniformly distributed in the square, find the expected travel distance of the ambulance.