

1. You and the bank play the following game: You flip  $n$  coins. If  $X$  of them come up heads, you receive  $2^X$  dollars.

(a)[1 point] You have to buy a ticket to play this game. What is the fair price of a ticket?

(b)[1 point] Prove: The probability that you break even (i.e. receive at least your ticket's worth) is exponentially small. (Hint: At least how many "heads" do you need to break even?)

(c)[1 point] Calculate the standard deviation of the variable  $2^X$ . Your answer should be a simple formula.

(d)[1 point] Show that your answer to (c) is asymptotically equal to an even simpler formula. Make it as simple as possible.

(e)[1 point] State what the (Weak) Law of Large Numbers would say about the variable  $2^X$ .

(f)[Bonus! +2 points] Prove that the (Weak) Law of Large Numbers does NOT hold for this variable.