1.(2 points) Prove: If X is nonnegative then, for all $a > 0$,

$$P(X \geq a) \leq \frac{E(X)}{a}.$$

(Note: We say that a random variable X is *nonnegative* if $X(\omega) \geq 0$ for all $\omega \in \Omega$.)

2. Let X be the number of prime factors of a randomly-chosen integer N. In this problem, we will determine whether the Poisson distribution would be a "good" choice for a distribution function for X.

(a)(1 point) Suppose that we have determined (through other means) that $E(X) = \ln \ln N$. Use this to define the Poisson distribution function $P(X = j)$ and show that it is, in fact, a distribution. Note: In order to receive credit, you cannot merely cite a result from class. You must show that this particular function is a distribution.

(b)(1 point) Use the function that you defined in part (a) to find μ (where μ is the parameter associated with the **Poisson random variable** X). Note: In order to receive credit, you cannot merely cite a result from class. You must actually compute μ.

(c)(1 point) Based on your answer to (b), do you think that the random variable X is a good candidate for using the Poisson distribution? Why or why not?

Bonus!(+2 points) Recall that we can construct a random graph by flipping a fair coin to determine whether each pair of vertices is connected by an edge. We define the *diameter* of a graph G to be the maximum distance between all pairs of vertices in G (where the "distance" is defined to be the shortest path between a pair of vertices). For example, a triangle has diameter 1, since every pair of vertices is connected by an edge, and a square has diameter 2, since one must travel along two edges to get between opposite corners. Prove that "almost all" random graphs have diameter 2. In other words, prove that if p_n is the probability that a random graph on n vertices has diameter 2, then $\lim_{n\to\infty} p_n = 1$.