MATH1 Day 17: The Derivative

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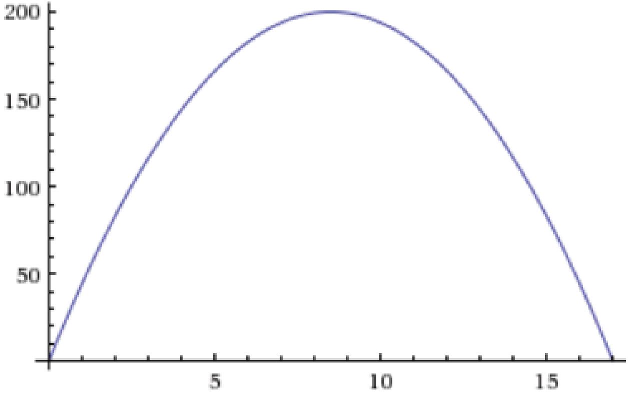
Intro to the Derivative

$$f(t) = -\frac{800}{289}(t^2 - 17t)$$



Intro to the Derivative

Group 1: Find the average speed on the interval $[5, 5.4]$

Group 2: Find the average speed on the interval $[5, 5.3]$

Group 3: Find the average speed on the interval $[5, 5.2]$

Group 4: Find the average speed on the interval $[5, 5.1]$

$f(5) = 166.09$, $f(5.1) = 168$, $f(5.2) = 169.85$, $f(5.3) = 171.65$, $f(5.4) = 173.4$

Intro to the Derivative: Definition

Definition

The derivative of f **at** a *is*

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided this limit exists. We read this " f *prime at* a*".*
This gives us instantaneous speed at a*, as well as the slope of the tangent line to the graph of* f *passing through* $(a, f(a))$

The derivative as a function: Definition

Definition

The derivative of f *is*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists. We read this " f prime "

Examples:

1. Find $f'(x)$ where $f(x) = 2x^2 - x$, and find the equaton of the tangent line at $x = -1, x = 0, x = 1$.

2. (If done with the above) Find $f'(x)$ where $f(x) = x^3 + x - 1$, and find the equaton of the tangent line at $x = 0$.

Examples:

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2. (If done with the above) Find $f'(x)$ where $f(x) = x^3 + x - 1$, and find the equaton of the tangent line at $x = 0$.

Examples:

1. Find $f'(x)$ where $f(x) = 2x^2 - x$, and find the equation of the tangent line at $x = -1, 0, 1$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 - (x+h) - (2 \cdot x^2 - x)}{h} =$$

$$\lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) - (x+h) - (2x^2 - x)}{h} =$$

$$\lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 - x - h - 2x^2 + x}{h} =$$

$$\lim_{h \to 0} \frac{2h^2 + 4hx - h}{h} = \lim_{h \to 0} (2h + 4x - 1) = 4x - 1$$

$f'(0) = -1, f(0) = 0$, so the tangent line at $x = 0$ is $y = -x$
$f'(1) = 3, f(1) = 1$, so the tangent line at $x = 1$ is $y - 1 = 3(x - 1)$
$f'(-1) = -5, f(-1) = 3$, so the tangent line at $x = -1$ is
$y - 3 = -5(x + 1)$

Examples:

2. Find $f'(x)$ where $f(x) = x^3 + x - 1$, and find the equaton of the tangent line at $x = 0$.

$$\lim_{h\to 0} \frac{(x+h)^3 + (x+h) - 1 - (x^3 + x - 1)}{h} =$$

$$\lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + x + h - 1 - x^3 - x + 1}{h} =$$

$$\lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + h}{h} = \lim_{h\to 0}(3x^2 + 3xh + h^2 + 1) = 3x^2 + 1$$

$f'(0) = 1, f(0) = -1$, so the tangent line at $x = 0$ is $y + 1 = 0$

Tangent line as an approximation of the graph

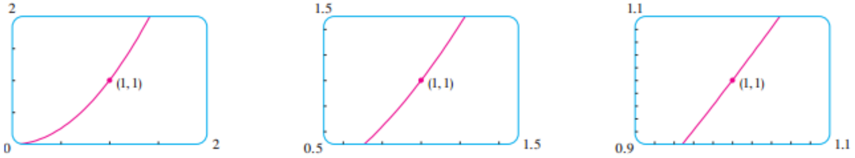

FIGURE 2 Zooming in toward the point (1, 1) on the parabola $y = x^2$

Figure: Taken from Stewart