# MATH1 Day 15: Continuity

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October 12, 2016

# Continuity at a point

## Definition

*A function $f$ is* **continuous at a number** *$a$ if*

$$\lim_{x \to a} f(x) = f(a).$$

**Note!** *If the function is not defined on one side of the point, we take* $\lim_{x \to a} f(x)$ *to be equal to the one-sided limit where the function is defined.*

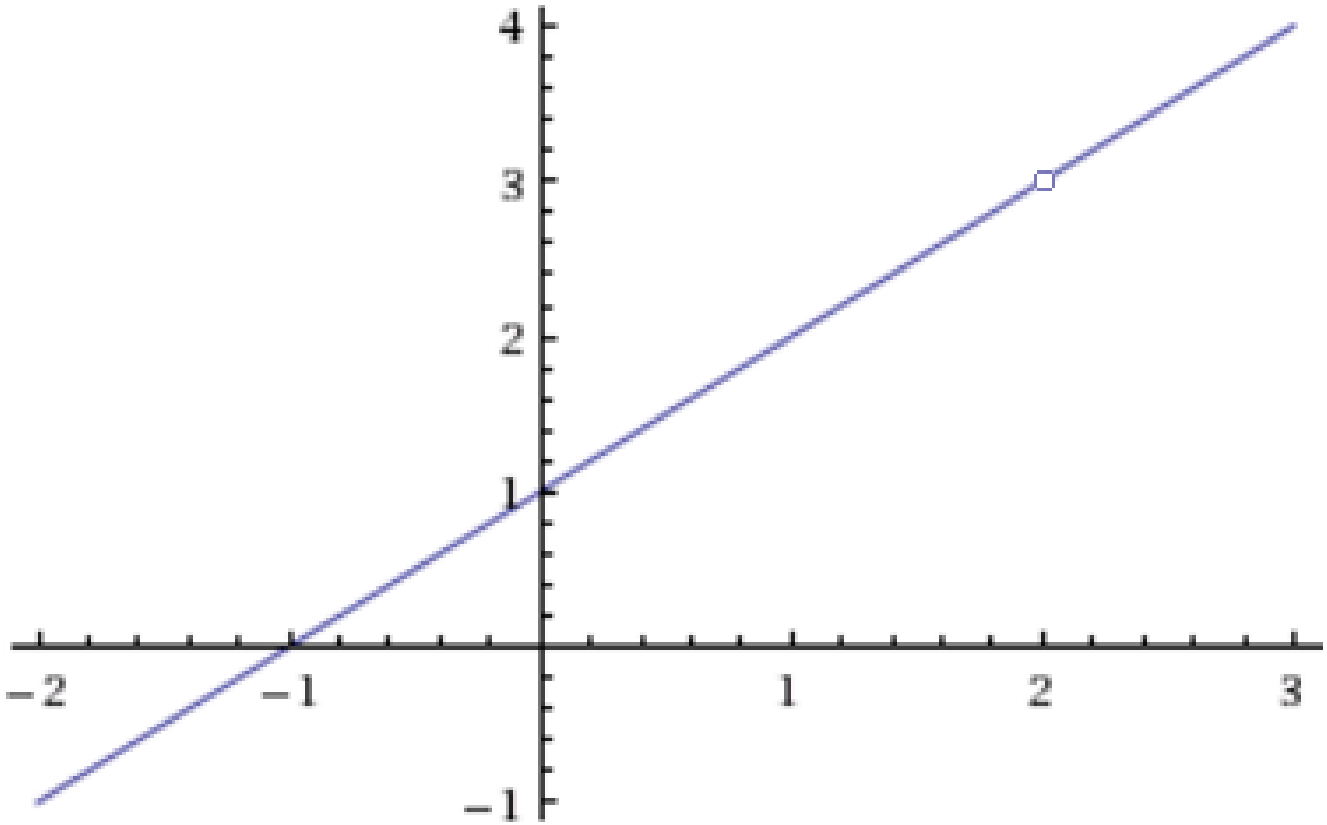
# Continuity on an interval

**Definition**

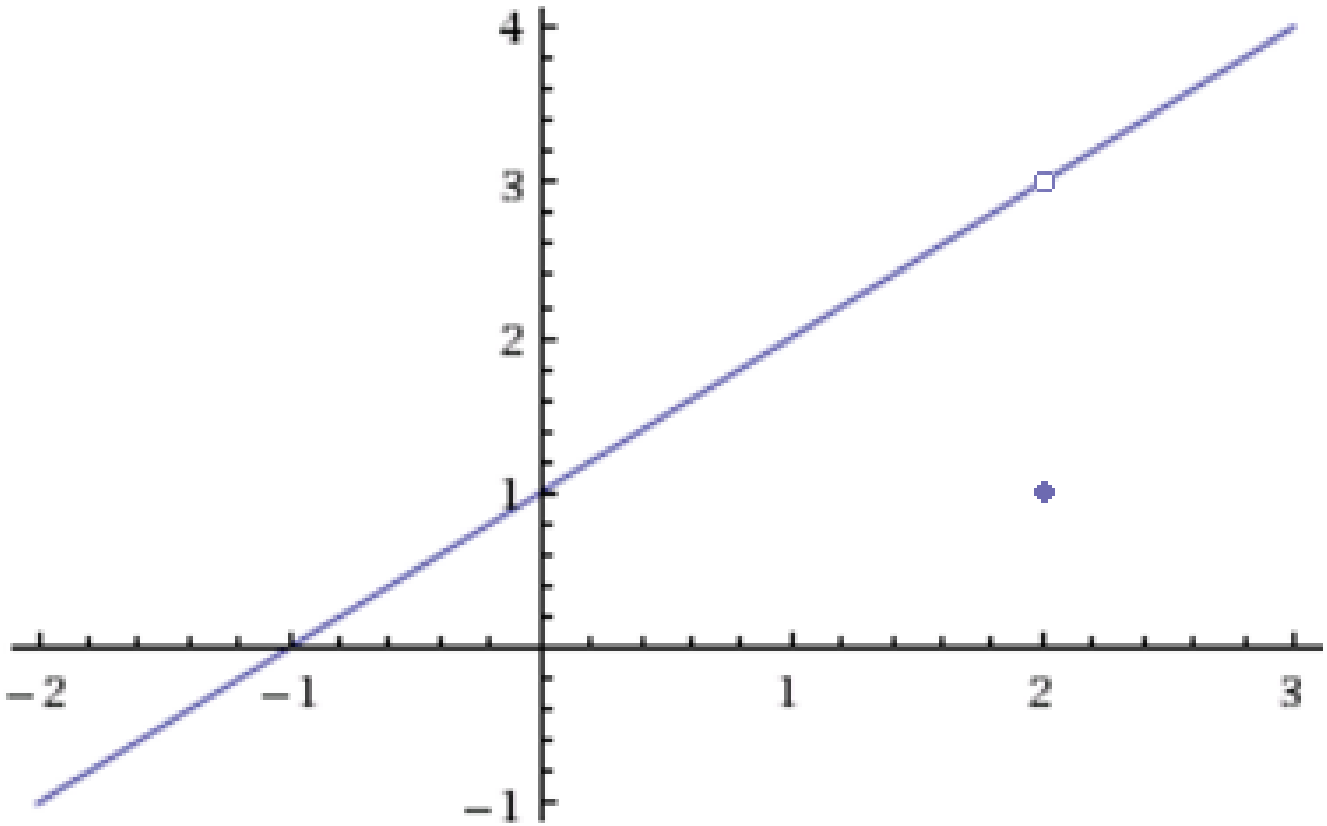
*A function $f$ is continuous on an interval if it is continuous at every number in the interval.*

Using the definition of continuity, answer the following question: Is $f(x)$ continuous at $x = 2$? Is $f(x)$ continuous on its domain?

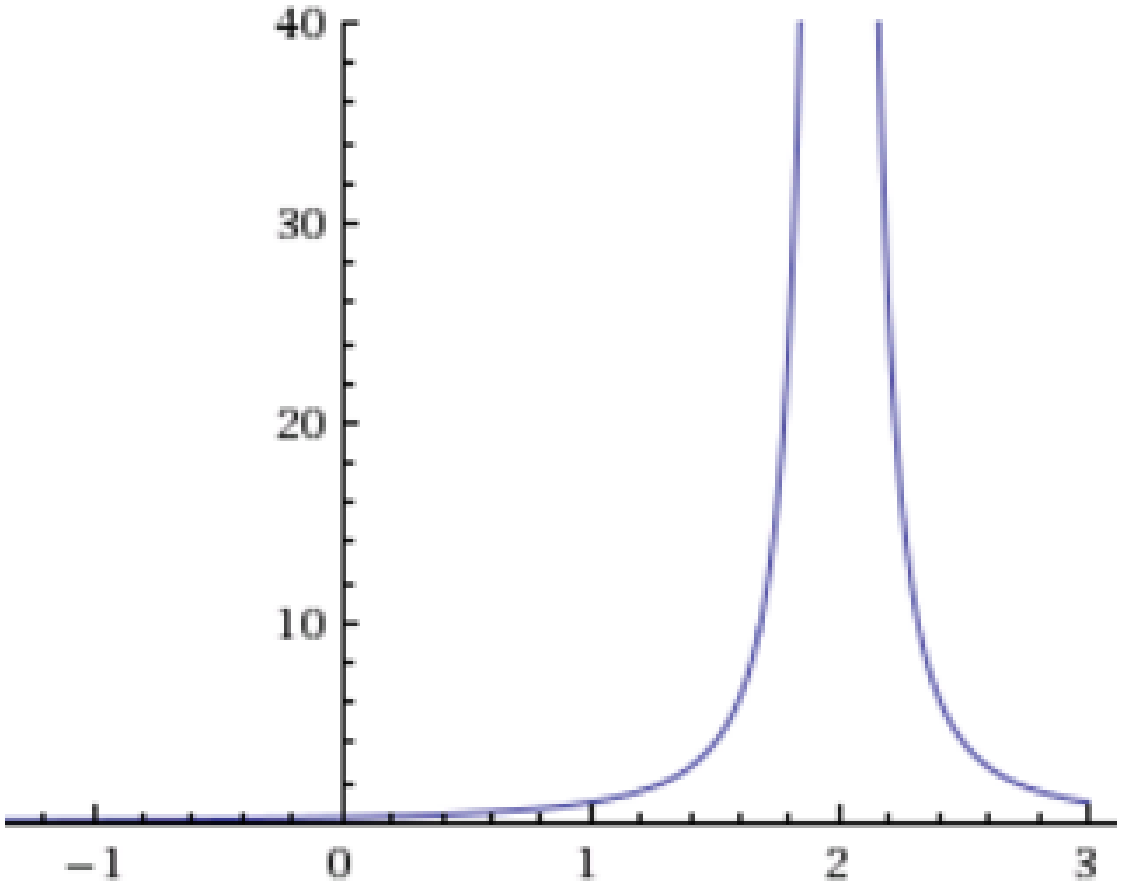
$f(x) = \frac{x^2-x-2}{x-2}$



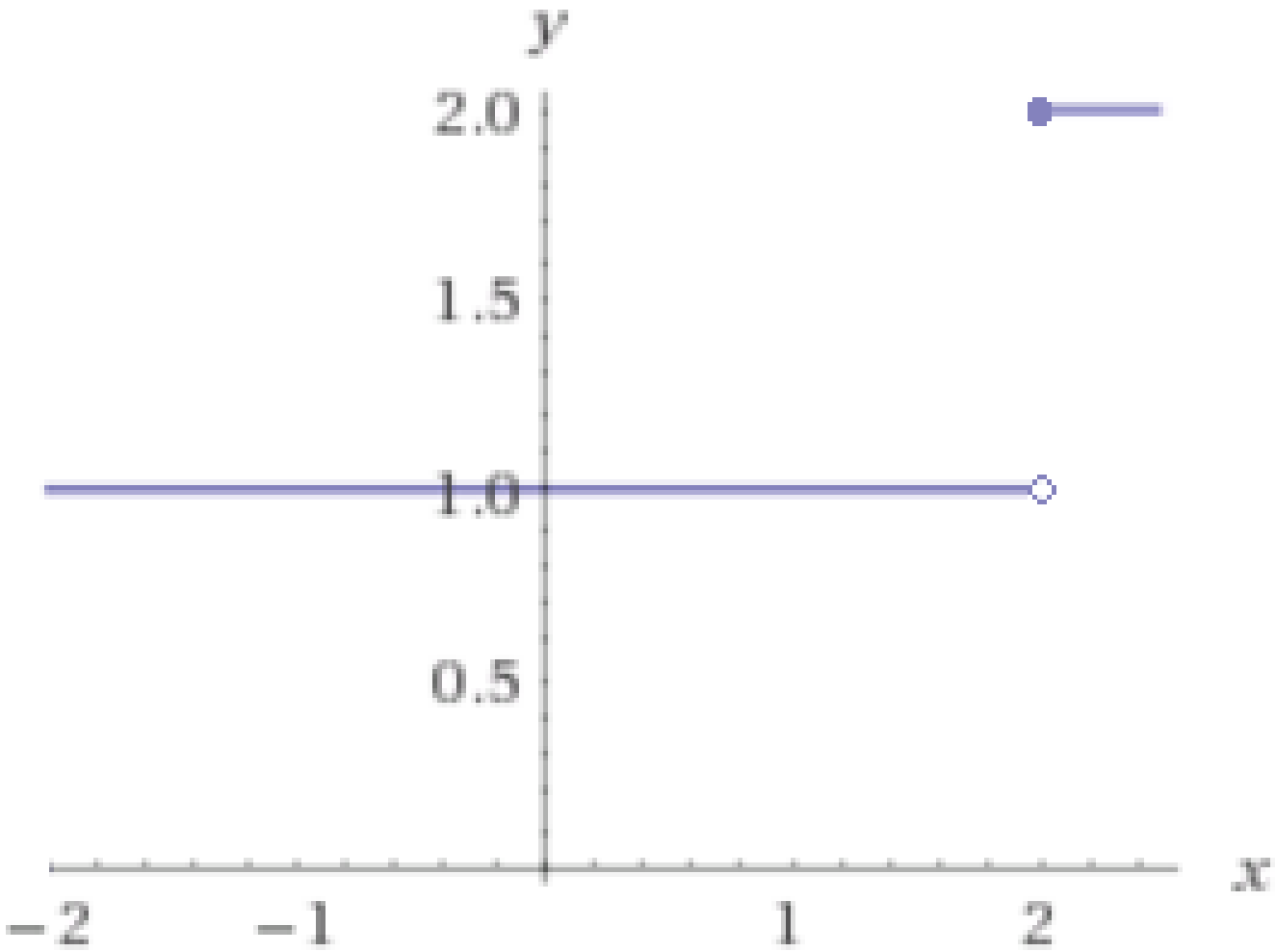
$$f(x) = \begin{cases} 1 & \text{if } x = 2 \\ \dfrac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \end{cases}$$



$f(x) = \frac{1}{(x-2)^2}$



$$f(x) = \begin{cases} 1 & \text{if } x < 2 \\ 2 & \text{if } x \geq 2 \end{cases}$$



# Continuity properties

**Theorem**

*If $f$ and $g$ are continuous at $a$ and if $c$ is a constant, then the following functions are also continuous at $a$:*

1. $f + g$
2. $fg$
3. $f - g$
4. $\frac{f}{g}$ *if* $g(a) \neq 0$
5. $cf$

# Continuity properties - part II

**Theorem**

*If $g$ is continuous at $a$ and $f$ is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at $a$.*