MATH1 Day 16: The Squeeze Theorem and the Derivative

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The Squeeze Theorem

Theorem

If $f(x) \leq g(x) \leq h(x)$ *when* x *is near* a *(except possibly at* a*), and*

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L,$$

then

$$\lim_{x \to a} g(x) = L.$$

The Squeeze Theorem Part II

1. If $f(x)$ is always between $x^2 + 2$ and $2x + 1$, find $\lim_{x\to 1} f(x)$.
2. Use the relation $\cos(x) \le \frac{\sin(x)}{x} \le 1$ to find $\lim_{x\to 0} \frac{\sin(x)}{x}$.
3. Let $f(x)$ be a function which satisfies $5x - 6 \le f(x) \le x^2 + 3x - 5$ for all $x \ge 0$. Compute $\lim_{x\to 1} f(x)$.

Example Show

$$\lim_{x\to 0} x^2 \arctan\left(\frac{1}{x}\right) = 0$$

We can't use the product property of limits, since $\lim_{x\to 0} \arctan\left(\frac{1}{x}\right)$ does not exists. However, we can use the Squeeze theorem: we know

$$-\frac{\pi}{2} \le \arctan\left(\frac{1}{x}\right) \le \frac{\pi}{2}$$

and

$$-x^2\frac{\pi}{2} \le x^2 \arctan\left(\frac{1}{x}\right) \le x^2\frac{\pi}{2}.$$

We know that $\lim_{x\to 0} x^2\frac{\pi}{2} = 0, \lim_{x\to 0}(-x^2\frac{\pi}{2}) = 0$, so by the Squeeze theorem,

$$\lim_{x\to 0} x^2 \arctan\left(\frac{1}{x}\right) = 0$$

The Squeeze Theorem Part III

Find

$$\lim_{x\to 3}(x-3)^2\sin\left(\frac{1}{\sqrt{x-3}}\right)$$

Solution:

We know that

$$-1 \leq \sin\left(\frac{1}{\sqrt{x-3}}\right) \leq 1$$

so

$$-(x-3)^2 \leq (x-3)^2 \sin\left(\frac{1}{\sqrt{x-3}}\right) \leq (x-3)^2.$$

Since $\lim_{x\to 3} -(x-3)^2 = 0$, $\lim_{x\to 3}(x-3)^2 = 0$, by the Squeeze theorem,

$$\lim_{x\to 3}(x-3)^2 \sin\left(\frac{1}{\sqrt{x-3}}\right) = 0$$

The Squeeze Theorem

Exercise 1. Find

$$\lim_{x\to 0} x^2 e^{\cos\left(\frac{2x+1}{x^2}\right)}$$

Exercise 2. Find

$$\lim_{x\to -3} (x+3)^2 \cos\left(\frac{2}{x+3}\right)$$

Exercise 1. Find

$$\lim_{x\to 0} x^2 e^{\cos\left(\frac{2x+1}{x^2}\right)}$$

Solution:

We know

$$-1 \leq \cos\left(\frac{2x+1}{x^2}\right) \leq 1$$

so

$$e^{-1} \leq e^{\cos\left(\frac{2x+1}{x^2}\right)} \leq e^1$$

$$x^2 e^{-1} \leq x^2 e^{\cos\left(\frac{2x+1}{x^2}\right)} \leq x^2 e^1$$

Since $\lim_{x\to 0} x^2 e^{-1} = 0$, $\lim_{x\to 0} (x^2 e) = 0$, by the Squeeze theorem

$\lim_{x\to 0} x^2 e^{\cos\left(\frac{2x+1}{x^2}\right)} = 0$

Exercise 2. Find

$$\lim_{x \to -3} (x+3)^2 \cos\left(\frac{2}{x+3}\right)$$

Solution:

We know that

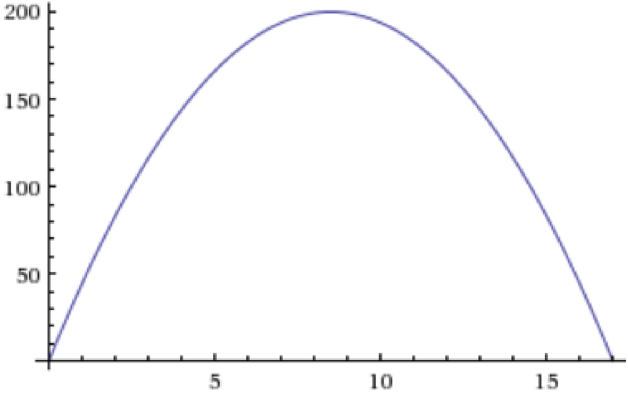
$$-1 \leq \cos\left(\frac{2}{x+3}\right) \leq 1$$

$$-(x+3)^2 \leq (x+3)^2 \cos\left(\frac{2}{x+3}\right) \leq (x+3)^2.$$

Since $\lim_{x \to -3} (x+3)^2 = 0$, $\lim_{x \to -3} -(x+3)^2 = 0$, by the Squeeze theorem, $\lim_{x \to -3} x^2 \cos\left(\frac{2}{x+3}\right) = 0$

Intro to the Derivative

$$f(t) = -\frac{800}{289}(x^2 - 17x)$$



Intro to the Derivative

Group 1: Find the average speed on the interval $[5, 5.4]$

Group 2: Find the average speed on the interval $[5, 5.3]$

Group 3: Find the average speed on the interval $[5, 5.2]$

Group 4: Find the average speed on the interval $[5, 5.1]$

$f(5) = 166.09$, $f(5.1) = 168$, $f(5.2) = 169.85$, $f(5.3) = 171.65$, $f(5.4) = 173.4$

Intro to the Derivative: Definition

Definition

The derivative of f **at** a *is*

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided this limit exists. We read this " f *prime at a".*