Week 1: need to know

How to find domains and ranges, operations on functions (addition, subtraction, multiplication, division, composition) , behaviors of functions (even/odd/ increasing/decreasing), library of functions (recognizing power, constant, linear, polynomial, rational, algebraic functions by formulas and graphs). Sequences: find terms of the sequence by the formula, identify formula by the first few terms of the sequence; behavior of sequences (increasing, decreasing, bounded: finding a bound)

Week 1 examples/exercises

Examples:

1. Product or composition?
$f(x) = x\ln(x), g(x) = \cos(x)\sin(3x), h(x) = \sqrt{3\ln(x)}, j(x) = \arcsin(3x^2) + \ln(3x^2), k(x) = \arcsin(3x^2)\ln(3x^2)$
2. A function is a rule that consists of three parts: a domain, a range, a formula. Let $f(x) = x^2$ be the function that gives the area of a square. Domain: values we can input in the function : positive real numbers. Range: values we can get out: positive real numbers. The formula is $f(x) = x^2$, but the rule itself also needs to mention the domain in some form (in this case, since we talk about areas, we know we can only input positive values).
Exercises: (a) if $f(x) = x^2, g(x) = \sqrt{x}, h(x) = x$, find the domain and range of $f \circ g, g \circ f, h/g$.
(b) Find the domain and range of $f(x) = \ln(3x+1)$ and its derivative $f'(x)$
3. Is $a_n = \frac{n+2}{n+3}$ increasing/decreasing/bounded? If it is bounded, find a bound.

Week 2: need to know

Using Lagrange interpolation formula; one-to-one functions; inverse functions: what they are both by formula and conceptually, how to find their domains and ranges, how to find the formula of an inverse function ; function transformations: given the domain and range of $f(x)$, finding the domain and range of $cf(ax+b)+d$ (also inverses); exponential and logarithmic functions (graphs: domains and ranges) and solving equations. Quadratic formula.

Week 2 examples/exercises

Examples:

1. Is $\sqrt{x-2}+5$ one-to-one? Is $(x-3)^2+5$ one-to-one of the interval $[1,3]$? How about on $[1,5]$?
2. Draw a one-to-one function. Can your sketch its inverse? Algebra: what is the inverse function of $\sqrt{x-2}+5$? What is the domain and range of the inverse function?
3. Function transformations: the order of transformations, domains and ranges.
   Exercises: let $f(x)$ have domain $[1,4]$ and range $[-2,5]$. Find the domain and range of the following functions, and specify the order of the transformations: $2f(-x+1), 3f^{-1}(x+3), -f(2x+1)$
4. Solve for x:
   $e^{3^x}=3^{e^x}, \ln(x+2)+\ln(x+3)=\ln(12), \ln(x+2)-\ln(x+3)=\ln(12)$

Week 3: need to know

Trig and inverse trig functions (how we get such functions and what they mean, recognizing and sketching their graphs, finding values of such functions, domains and ranges, solving equations).

Week 3 examples/exercises

Examples:

1. Sketch the graph of $3\sin(2x+\frac{\pi}{2})$ on the interval $[-2\pi, 2\pi]$. What is the amplitude and period of this function?
2. Compute the following values:
   $\cos(-\frac{2\pi}{3}), \arccos(\cos(-\frac{2\pi}{3})), \arcsin(\sin(-\frac{\pi}{8})),$
   $\tan(\arccos(\frac{4}{9})), \sin(\arccos(\frac{x+3}{x+5}))$
3. Find the domain and range of $\arccos(-x+3), 3\arcsin(2x+4)$
4. Solve for x: $\cos(x)=0, \arccos(x)=\frac{\pi}{3}$, $\tan(3x+\frac{\pi}{4})=1$,
   $\tan(3x+\frac{\pi}{4})=-1$

Week 4: need to know

Convergence and behaviour of sequences (recognizing when a function converges and to what limit; can a function converge and be bounded/unbounded? can a function converge and be monotone/not be monotone? does a monotone bounded sequence converge?), the squeeze theorem for sequences ($\frac{-1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}$, so $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$); for what values does r^n converge? reading limits of function from their graphs, recognizing asymptotes.

Week 4 examples/exercises

Examples:

1. Which of the following sequences converges, and to what limit?
   $a_n = \frac{n^2+3n}{(2n-1)(5n-6)}$, $b_n = \frac{-3^n}{\pi^n}$,
   $c_n = \ln(\frac{3n+1}{2n^2-3})$, $d_n = e^{-n+2}$, $k_n = \arctan(\frac{4n^2}{3n-2})$
2. Find the horizontal and vertical asymptotes of the following functions:
   $f(x) = \frac{x-1}{x+1}$, $g(x) = \frac{x-3}{x^2+x-12}$, $h(x) = \frac{x^2+2x+1}{x^2-2x-3}$

Week 5: need to know

Finding limits of elementary functions (algebraic, trig and inverse trig, exponential, logarithmic, both by looking at their graphs and by their formula), properties of limits (finding limits of combinations of functions), the squeeze theorem; continuity of functions: recognizing the intervals where a function is continuous (both by looking at the graph and formula), recognizing types of discontinuities (graphs and formulas)

Week 5 examples/exercises

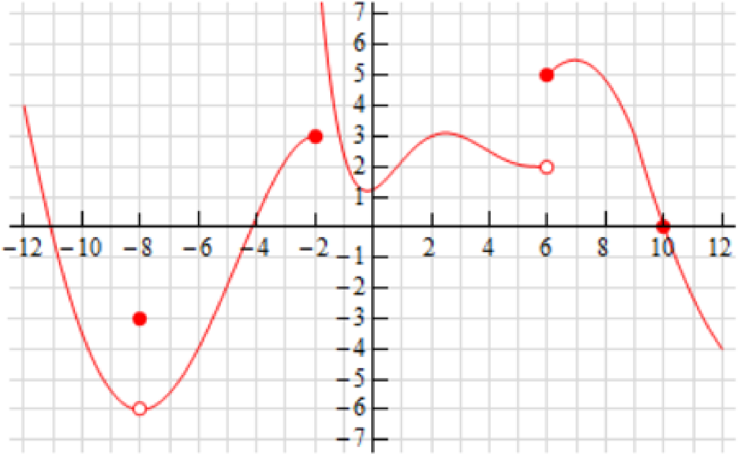
Examples:

1. Find $\lim_{x \to 2} (x-2)^2 \sin\left(\frac{x-1}{(x-2)^2}\right)$
2. On what intervals are the following functions continuous: $f(x) = \ln(x^2 + x - 12)$, $g(x) = \frac{(x-2)^2}{(x-2)(x+10)}$, $h(x) = \frac{\sqrt{x+3}}{\sqrt[3]{3-x}}$.

For each of the following functions, determine
I. where it is discontinuous, and what types of discontinuities those are
II. the intervals on which the function is continuous
III. $\lim_{x \to \infty} f\left(\frac{-12x^2 + 1}{2x^2 + 10x + 1}\right)$

Week 5 examples/exercises continued

(a) If $f(x)$ is given by the following graph



Week 5 examples/exercises continued

(b) If f is defined piecewise the following way:

$$f(x) = \begin{cases} \ln(-x) & \text{if } x < 0 \\ \dfrac{1}{x} - \dfrac{1}{3} & \text{if } 0 < x < 3 \\ 5 & \text{if } x = 3 \\ \dfrac{x-3}{(x-5)(x-10)} & \text{if } x > 3 \end{cases}$$

Week 6: need to know

How to describe the derivative as instantaneous speed, the rate of change of $f(x)$ w.r.t. x, and the slope of the tangent line at a point; what do $\frac{d}{dx}$, $\frac{d\,f(x)}{dx}$, $\frac{d}{dx}$, $\frac{dt^2}{dt}$, $\frac{d}{dx}f(x)$ mean; how to find derivatives using the limit definiton of the derivative; formulas for derivatives: sum/difference formula, product formula, derivatives of polynomials, derivative of e^x.

Week 6 examples/exercises

Examples:

1. If the derivative gives the slope of the tangent line, and the tangent line is the best linear approximation of the graph at a point (so as we zoom in, the graph on an inteval near the point looks more and more like the tangent line at that point), where would we not have a derivative? (Hint: think of situations we have encountered during implicit differentiation)
2. Using ONLY the limit definition of the derivative , find $f'(x)$ when $f(x) = \frac{x}{x+3}$; $g'(x)$ for $g(x) = x^2 + 3x + 2$.

Week 7: need to know

Derivatives of trig functions; the formulas using the product and quotient rule; finding iterated derivatives.

Week 7 examples/exercises

Examples: Find the second derivatives of the following functions:

1. $f(x) = \sin(x) + e^x$,
2. $g(x) = \frac{\cos(x)}{\sin(x)}$,
3. $h(x) = \tan(x)\sin(x)$,
4. $j(x) = xe^x$,
5. $k(x) = \frac{h(x)}{j(x)}$

Week 8: need to know

The formula and how to use the chain rule; implicit differentiation: finding $\frac{dy}{dx}$, recognizing at what point of the curve we have a tangent line and finding the equation of the tangent line; derivatives of inverse trig functions: $\arcsin, \arccos, \arctan$; derivatives or exponential and logarithmic functions

Week 8 examples/exercises

Examples:

1. Find the derivative w.r.t. x of the following functions:
   1. $f(x) = e^{\sin(x)}$,
   2. $g(x) = \frac{\cos(x^2)}{\arctan(x)}$,
   3. $h(x) = x^2 \ln(x^2)$,
   4. $j(x) = \arcsin(2^x)$
   5. $k(x) = \frac{h(x)}{j(x)}$
2. Find $\frac{dy}{dx}$ for the following curves, and the equation of the tangent line at $(0, 1)$ (if it exists)
   1. $\ln(y) + xy = x^2$,
   2. $x^3 + y^2 - xy = 0$,

Week 9: need to know

The formula and how to use Newton's method, linear approximation, Taylor approximation.

Week 9 examples/exercises

Examples:

1. Draw a picture illustrating Newton's method, and a picture where Newton's method would probably not work very well.
2. What is the defference between Newton's method and linear approximation?
3. What is the point of Taylor approximation?
4. Let $f(x) = x^3 + 3x + 2$. Starting with $x_0 = -1$, do two iterations of Newton's method to approximate x_2 as a fraction.
5. Let $f(x) = e^{2x}$. Find its fourth Taylor polynomial centered at $x = 0$
6. Let $g(x) = (x+4)(x^{99} - 4)$. Find its 99th and 100th Taylor approximations centered at $x = 0$.
7. Let $h(x) = \sqrt{x-2}$. Find its third Taylor polynomial centered at $x = 1$