Practice problems review I

Exercise 1: domains and ranges; inverse functions

(1) Let $f(x) = \sin(x), g(x) = \arcsin(x)$. What are the domains and ranges of $f, g, f \circ g$ and $g \circ f$?
(2) Let $f(x) = x^2, g(x) = \sqrt{x+1}$. What are the domains and ranges of $f, g, f \circ g$ and $g \circ f$?
(3) Let $f(x) = x + 2, g(x) = \sqrt{x+2}$. What are the domains and ranges of $\frac{f}{g}$ and $\frac{g}{f}$?
(4) Let $f(x) = e^x, g(x) = \ln(x+3)$. What are the domains and ranges of $f \circ g$ and $g \circ f$?

Exercise 2: logarithm and exponential equations. Solve for x:

(1) $\ln(x+2) + \ln(x-2) = \ln(6)$
(2) $\ln(x+3) - \ln(x-3) = \ln(5)$
(3) $2^{3x+1} = 4^x$
(4) $2^{e^x} = e^{2^x}$

Exercise 3: library of functions. Consider the following classes of functions: linear, power, polynomial, rational, algebraic. For each of the following functions, write down which classes it belongs to and which classes it doesn't belong to (all five classes should be mentioned).

(1) $f(x) = 1$
(2) $g(x) = x^2 + 1$
(3) $h(x) = \sqrt{x^3}$
(4) $k(x) = \dfrac{x+1}{x+1}$
(5) $f(x) = x^{3\pi}$

Exercise 4: Let $f(x)$ be a function with domain $[-2, 3]$ and range $[0, 8]$. What are the domains and ranges of the following functions?

(1) $-f(-x-1)$
(2) $3f(2x+1)$
(3) $4f^{-1}(-x) + 1$

Exercise 5: True/False Are the following statements true or false?

(1) $\sin(x)$ is an even function
(2) $\sin(x)$ is an odd function
(3) $\cos(x)$ is an even function
(4) $\cos(x)$ is an odd function
(5) e^x is an increasing function
(6) $\ln(x)$ is a decreasing function

(7) The sequence $a_n = \frac{2n+1}{3n}$ is bounded by $2/3$

(8) The function $\frac{3x^2}{5x-1}$ is even

(9) The function $(x-5)^2+5$ is one-to-one on the interval $[-1,5]$

(10) The function $(x-5)^2+5$ is one-to-one on the interval $[0,7]$