DIFFERENTIATION WORKSHEET V

Find the following derivatives:

(1) $\frac{d}{dx}(\ln\left(\frac{3x+2}{2x-1}\right)) = \frac{1}{\frac{3x+2}{2x-1}}\cdot\frac{3(2x-1)-(3x+2)2}{(2x-1)^2} = \frac{2x-1}{3x+2}\cdot\frac{-7}{(2x-1)^2} = \frac{-7}{(3x+2)(2x-1)}$

We have used the chain rule and the quotient rule: for the chain rule we let $f(x) = \ln(x)$ so $f'(x) = \frac{1}{x}$, and $g(x) = \frac{3x+2}{2x-1}$ so $g'(x) = \frac{3(2x-1)-(3x+2)2}{(2x-1)^2}$ by the quotient rule

(2) $\frac{d}{d(3x^2)}(\arctan(3x^2)\log_5(3x^2)) = \frac{d\,\arctan(3x^2)}{d(3x^2)}\log_5(3x^2)) + \arctan(3x^2)\frac{d\log_5(3x^2))}{d(3x^2)} = \frac{1}{1+(3x^2)^2}\log_5(3x^2)) + \arctan(3x^2)\frac{1}{\ln(5)3x^2}$

We used the product rule

(3) $\frac{d}{dy}\left(\frac{y}{5^y}\right) = \frac{1\cdot 5^y - y\cdot(\ln(5)5^y)}{(5^y)^2} = \frac{1-y\ln(5)}{5^y}$ by the quotient rule

(4) $(\sin(3x\arccos(x)))' = \cos(3x\arccos(x))\left(3\arccos(x) - \frac{3x}{\sqrt{1-x^2}}\right)$

We have used the chain rule and the product rule: for the chain rule we let $f(x) = \sin(x)$ so $f'(x) = \cos(x)$, and $g(x) = 3x\arccos(x))$ so $g'(x) = 3\arccos(x) + 3x\frac{-1}{\sqrt{1-x^2}}$ by the product rule

(5) $(3\sqrt[3]{2x^3+5})' = 3\cdot\frac{1}{3}(2x^3+5)^{-2/3}6x$

We have used the chain rule: let $f(x) = x^{1/3}$ so $f'(x) = \frac{1}{3}x^{-2/3}$, and $g(x) = 2x^3+5$ so $g'(x) = 6x$

Calculate $\frac{dy}{dx}$ for the folowing curves:

(1) $x^2 + xy + y^2 = 0$

$$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(0)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0$$

$$2x + (y + x\frac{dy}{dx}) + 2y\frac{dy}{dx} = 0 \text{ (the term in the middle follows by the product rule)}$$

$$2x + y = -x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$2x + y = (-x - 2y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x + y}{-x - 2y}$$

(2) $\cos(x + y) = x^2 + 3y$

$$\frac{d}{dx}\cos(x + y) = \frac{d}{dx}(x^2 + 3y)$$

$$-\sin(x + y)\left(1 + \frac{dy}{dx}\right) = 2x + 3\frac{dy}{dx}$$

$$-\sin(x + y) - \sin(x + y)\frac{dy}{dx} = 2x + 3\frac{dy}{dx}$$

$$-\sin(x + y) - 2x = 3\frac{dy}{dx} + \sin(x + y)\frac{dy}{dx}$$

$$-\sin(x + y) - 2x = (3 + \sin(x + y))\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-\sin(x + y) - 2x}{3 + \sin(x + y)}$$

(3) $\ln(y) = y^3 + x^3$

$$\frac{d}{dx}\ln(y) = \frac{d}{dx}(y^3 + x^3)$$

$$\frac{d\ln(y)}{dy}\frac{dy}{dx} = \frac{d}{dx}(y^3) + \frac{d}{dx}x^3$$

$$\frac{1}{y}\frac{dy}{dx} = 3y^2\frac{dy}{dx} + 3x^2$$

$$\frac{1}{y}\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 3x^2$$

$$\left(\frac{1}{y} - 3y^2\right)\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{\frac{1}{y} - 3y^2}$$